Finite Observer Consensus as a Reconstruction Principle: Normal Forms, the Standard Model Lie Type, and a Route to the Einstein Field Equation

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Abstract

Observer Patch Holography asks whether a timeless, self-referential universe can be reconstructed from consistency among finite self-reading patches. Five layers are distinct: a self-referential closure hypothesis, three finite axioms, named repair and response laws, exact finite consequences, and physical realization maps. The patches carry state, records, overlap read-back, and an oriented twelve-port boundary. We prove conditions for a canonical observable normal form, stable under refinement and independent of repair schedule. Read on transition distributions, the third axiom yields a conditional finite four-law package once the common source reference and repair-fibre attachments are supplied. The second law is then a data-processing inequality for repair; the physical energy-clock calibration remains a named premise. Complete reversible port response with endogenous overlap transport forces the compact Lie-algebra type $\mathfrak{u}(1) \oplus \mathfrak{su}(2) \oplus \mathfrak{su}(3)$ without an ambient gauge group. A matter premise gives an anomaly-free fifteen-state exterior module with a $\mathbb{Z}_6$ kernel. The spherical support carries Lorentz kinematics. On a distinct signed-record branch, the normalized repair response has an exact rank-three quotient and an abstract continuous Euclidean completion. Physical position and overlap gluing remain premises. Given one physical refinement tower with the displayed stress, entropy, continuum, and scale properties, null tomography and the Bianchi identity yield the Einstein field equation up to a cosmological constant. An icosahedral theorem fixes the first allowed anisotropic harmonic at angular rank six. On the registered primitive twelve-port propagation branch, the quartic coefficient fixes both sixth-order coefficients and the rotated rank-six template. This frozen prospective prediction differs from the zero-coefficient minimal locally Lorentz-invariant Standard Model plus General Relativity baseline. Its physical sector, coherent frame, exclusivity, continuum tower, and laboratory attachments enter as premises.

Keywords: foundations of physics; Observer Patch Holography; finite reconstruction; quantum records; Einstein field equation; gauge structure.

1 Introduction and claim boundary

The program begins from one overarching aim: describe physical reality as a timeless, closed, self-referential structure whose content is fixed by consistency and needs no external cause. The closure hypothesis supplies that foundational picture. The finite observer-patch architecture, its axioms, and its physical consequences provide a candidate realization whose claims can be separated and tested.

Quantum probability, Lorentzian kinematics, gravitational dynamics, and gauge structure enter standard theories through distinct mathematical inputs [15, 16, 17, 40, 49]. Observer Patch Holography (OPH) asks which structures follow from a specified finite patch system whose components retain records, compare overlap readbacks, and repair disagreement. The source object is narrower than generic observer consistency: it includes a declared twelve-port carrier, complete reversible response, endogenous overlap transport, explicit repair hypotheses, a finite algebra-state representation, and the interpretation maps stated with each physical conclusion.

The logical structure has five layers. First, the closure hypothesis says that the physical universe is a fixed point of its own internal description and repair; it is the overarching structural premise. Second, A1 fixes the finite carrier and complete reversible-response tangent, A2 fixes agreement and endogenous overlap transport, and A3 selects the least-informative state left by those constraints. Together they state one finite realization of the closure picture. Third, named repair, response, and propagation laws act on the carrier. The axioms do not silently supply termination, confluence, or a physical kinetic operator. Fourth, finite theorems and certified computations state what follows from each displayed premise set. Fifth, physical realization maps identify finite records, currents, clocks, stress, and scales with laboratory or spacetime objects. The Lorentz statement is a group-theoretic consequence of the spherical support. The Einstein statement is tensor completion from a realized null balance, Ward conservation, the Bianchi identity, and an independent scale.

The closure reading orders these layers. Observers arise inside the structure, recover the simulator architecture from their public records, and, within their emergent subjective time, construct the recovered hardware and protocol. The reconstruction, the construction, and the universe that contains both are parts of one timeless fixed point. This observer process expresses the closure hypothesis in subjective time; it is not an independent source of A1–A3. Those axioms are the declared finite conditions under which this realization closes consistently. No prior temporal or causal derivation of them is required.

OPH is developed through a linked paper and artifact stack. This paper gives its synthesis and claim boundary; companion work supplies the finite formalization, exact-computation certificates, consensus construction, screen architecture, and simulation evidence [1, 5, 6, 7, 8, 11, 10, 2]. Together these sources specify a proposed observer-patch architecture and its conditional physical consequences. Each conclusion retains its own premises and verification artifacts, which readers can inspect directly.

Table 1 gives the seven status descriptions used throughout.

Throughout, "source" means computed from the declared finite source specification, including every explicitly named source law, with no measured physical input. "Source-only" marks a quantity whose entire dependency cone is of that kind. A physical realization map is not source-only merely because the finite branch calculation is target-clean.

The distinction is mathematical: a finite implication and a physical realization of its antecedents are different claims. Machine-readable receipts record that distinction and reject a physical verdict when a required map is absent.

Boundary 1.1. This paper claims no completed derivation of particle masses, no source-only determination of the fine-structure constant, no completed cosmology, and no claim that formal

Table 1: Claim-status vocabulary used in the manuscript.

Finite theorem	A deductive result proved from displayed premises on a stated finite domain. An exhaustive computation counts as part of the proof only with a specified domain and a soundness argument.
Certified finite computation	An executable witness whose inputs, controls, and output are fixed by a reproducible certificate; execution alone does not promote its interpretation to a theorem.
Reconstruction implication	An exact implication whose antecedents include displayed geometric or physical premises.
Open realization map	A required map from a finite object to a physical event, current, field, clock, scale, or continuum structure for which no construction is asserted.
Diagnostic	A certified computation compared against measured values that it consumes; never counted as a prediction.
Registered conditional test	A diagnostic with a custody-bound target definition, kill band, precision floor, and decision rule fixed before comparison.
Frozen prospective branch prediction	A source calculation and decision rule fixed before any eligible comparison, conditional on a named physical branch. Failure rejects that branch; its scope extends to the whole framework only if the branch is proved forced and exclusive.

verification establishes physical truth. Quantitative surfaces that consume measured values are labeled diagnostics. The primitive twelve-port propagation branch is a frozen prospective conditional prediction. Section 12 states its decision rule and the physical premises that set the scope of any verdict.

Definition 1.2 (Physical realization map)**.** A physical realization map assigns finite public records, event data, currents, modular generators, and scales to physical records, spacetime events, laboratory currents, clocks, and units. It must preserve the algebraic operations, overlap restrictions, causal order, normalizations, and refinement relations consumed by the conclusion in question. A resemblance of spectra, dimensions, or symmetry names is insufficient.

Throughout this paper, a *contract* is a named, typed theorem interface. It records the inputs, domains, maps, normalization conventions, premises, outputs, and realization obligations for one implication. The term keeps the full dependency surface visible and introduces no additional physical law. Thus a transaction contract, matter contract, or Einstein contract names the specific interface consumed by that result.

1.1 Principal contributions

The observer-indexed normal-form framework separates existence, observable determination, schedule independence, stability, and computational cost. It gives a canonical partial normalizer, a sharp two-output stability bound, an accumulated refinement bound, and complexity barriers for succinct boundary problems. On the declared transactional repair branch, semantic dependency closure and revalidation yield the local diamond and hence a schedule-independent public record.

The declared twelve-port repair mean also gives a conditional local carrier theorem. Its centered equal-port response kernel selects a rank-three projector after slow-response normalization and the infinite-step limit. The projector range is intrinsic to the twelve labeled ports. The signed cumulative port-record/load module maps onto $\mathbb{Z}^6$, while the conservative seam currents span its even-sum submodule D_6. In the pullback limiting Gram metric, both modules are dense in an abstract continuous three-dimensional Euclidean translation carrier. This is a local carrier-position readback, canonical up to port-label-preserving isometry. Cumulative records act by exact internal translations, and the proper carrier maps act isometrically. Selection of the repair law, the commutative record module, and the Gram readback topology are premises. Overlap/refinement gluing, physical scale, and physical-space identification are open.

Complete reversible response gives a faithful compact twelve-dimensional unitary current algebra. Endogenous overlap transport makes every proper carrier action inner on that same algebra. Its one-dimensional fixed space and compact-simple classification force $\mathfrak{u}(1) \oplus \mathfrak{su}(2) \oplus \mathfrak{su}(3)$ without an ambient gauge-group premise. A declared four-band matrix map is an exact conditional witness for this type. Under a separate trace-balanced matter contract, the exterior module gives a faithful image with $\mathbb{Z}_6$ cover kernel and an anomaly-free fifteen-state representation. The source contains no selection of the matrix current, physical matter action, global quotient, or laboratory current; the physical reading consumes these as premises.

When the completed finite record surface is represented by a declared algebra-state pair, its public projectors carry exact projection-event, conditioning, expectation, and Clauser–Horne–Shimony–Holt identities. The geometry branch distinguishes observer-velocity kinematics from event geometry. Its Einstein result is a reconstruction implication whose modular, stress, entropy, area, Ward, continuum, and scale premises are displayed in one contract.

Conditional on a supplied finite algebra, reference, and interface, the dynamical layer has sharp normal forms. Every pointwise-continuous one-parameter star-automorphism group of the finite private algebra is blockwise unitary conjugation generated by one time-independent self-adjoint Hamiltonian per Wedderburn block, unique up to a real scalar. Every nonnegative normalized effect valuation that is additive on coexisting effects equals the trace against a unique density operator, in every finite dimension including two, with a sharp-web countermodel and exact controls showing that the declared finite unsharp and real source webs underdetermine the global weight. Applying the declared two-dimensional representation to source-realized gauge labels gives a noncommuting algebraic pair that generates the missing Pauli-Y projector and completes two-dimensional tomography, but the source has no phase operation or outcome receipt. The path law of the realized source chain is the exponential tilt of the step-uniform reference, itself the unique reference invariant under independent target scrambling at fixed source, by the log-transition action, unique up to an additive constant and a multiplier rescaling; a finite Legendre bridge makes discrete Euler–Lagrange transport equivalent to one step of the discrete Hamilton flow. The same complete binary history law admits distinct strictly convex real enrichments and Hamiltonians, so it does not select the required velocity curvature. A declared equal-weight bracket on the twenty oriented faces is exactly $60R_{13}$ and fails Jacobi in 240 coordinates. Over the complete cone of invariant carrier metrics (the commutant of the port action is exactly four-dimensional), the certified phase diagram excludes P outright and selects G as the unique nearest compact family on every sector-balanced metric and on the whole box $\beta/\delta \in [1/50, 6]$; the mirror family F occupies a nonempty region across the exact three-scale tie surface, and Galois conjugation of $\sqrt{5}$ with the sector swap exchanges the two mirror distances, and the three exposed coordinate edit norms agree with the balanced verdict. Independently, ad-invariance reduces the carrier-form space from three weights to two for each color-bearing F/G bracket, with exact conjugate $\sqrt{5}$ relations, while the P control stays two-to-two. These are finite discriminators, not source selection of a bracket, repair law, or couplings.

The quantitative section proves the positive-chamber Koide identity and the capacity-transfer sign, states the conditional tau test and charged-lepton diagnostic, and derives the named primitive twelve-port propagation branch. That branch fixes the two sixth-order coefficients and the rotated angular-rank-six template from the quartic coefficient. Its custody record excludes every data product used by the related exploratory cosmic-microwave-background search.

Operational quantum reconstructions [15, 16, 17], relational and modular accounts of physical facts and time [29, 30], thermodynamic routes to gravitational dynamics [40, 41], and compact-group reconstruction [48, 49] provide the principal comparison points. Born and Lüders rules, Tsirelson's inequality, conformal–Lorentz isomorphisms, Tannaka reconstruction, and the familiar one-generation exterior representation are used as established ingredients. The originality asserted here is limited to the observer-fiber normalizer with sharp refinement control, the transaction contract yielding the local diamond, the A1–A2 forcing theorem with its fixed-space exclusion, the conditional four-band witness and exhaustive exterior selection, the nine-direction null design with exact determinant and decoder constants, the frozen coefficient-linked primitive-port prediction, the finite effect-valuation representation with its sharp- and unsharp-web countermodels and exact phase-lift boundary, the converse ladder that makes blockwise Hamiltonian generation a theorem, the derived log-transition action with its real-Legendre non-identifiability theorem, the oriented-face compact-family discriminator with its complete invariant-metric phase diagram, and the F/G invariant-form dimension-drop theorem.

1.2 The three axioms

The model uses the following three mathematical postulates.

A1 (oriented twelve-port observer screen). In plain language, the model assigns every carrier a finite observer screen with local state, readback, records, repair moves, checkpoints, typed overlaps, and an oriented twelve-port boundary. Concisely, for each regulator r there is a typed object

$$\mathfrak{N}_r = (\mathcal{P}_r, \mathcal{A}_r, \mathcal{R}_r, \mathcal{I}_r, \mathcal{U}_r, \mathcal{C}_r, N_r, S_r, b_r)$$

whose carrier boundary $K_{r,i} = (P_{r,i}, E_{r,i}, F_{r,i}, o_{r,i})$ has $|P| = 12$, $|E| = 30$, $|F| = 20$, and the oriented icosahedral incidence relations. The bridge $b_r : N_r \to S_r$ carries a designated oriented two-cycle to the fundamental class of the spherical support, and the typed maps commute with refinement. For each complete carrier, put $V_{r,i} = \mathbb{R}^{P_{r,i}}$. A1 supplies a finite-dimensional unitary space $H_{r,i}$ and an injective real-linear response derivative

$$D_{r,i} : V_{r,i} \longrightarrow \mathfrak{u}(H_{r,i})$$

whose image $\mathfrak{g}_{r,i} = D_{r,i}(V_{r,i})$ is closed under commutators. The primitive port probes span $V_{r,i}$, their ordered compositions generate every accepted infinitesimal reversible response, and no public response direction is omitted. The pairing

$$\langle v, w\rangle_D = -\operatorname{Tr}(D_{r,i}(v)D_{r,i}(w))$$

is positive definite. The response data and their completeness commute with refinement, and

$$G^0_{D,r,i} = \langle \exp(tD_{r,i}(v)) : t \in \mathbb{R},\ v \in V_{r,i}\rangle^0$$

is the connected response group. A1 constrains the finite carrier, federation, operational interfaces, spherical-support tower, and complete public reversible-response tangent. It does not imply semantic agreement, repair termination, confluence, a particular matrix response, an inverse-port law, a compact Lie type, a global group, or a laboratory identification [4].

A2 (observer agreement). In plain language, observers assign the same operational meaning to accepted data on their shared boundary. Every proper recharting of a complete carrier is implemented by their own reversible overlap response. Formally, the interpretation functor

$$\mathcal{J}_r : \mathsf{Data}_r \longrightarrow \mathsf{Meaning}_r$$

is natural under every visible restriction, recharting, seam translation, higher-overlap map, federation map, and refinement map. On an overlap O,

$$\mathcal{J}_O(\mathrm{res}_{P\to O}\, d_P) = \mathcal{J}_O(\tau_{Q\to P}\, \mathrm{res}_{Q\to O}\, d_Q).$$

The accepted reversible overlap transports form a groupoid $\mathcal{O}_r$. For a complete carrier chart o, let $\mathrm{Hol}_r(o)$ be its closed overlap paths. A2 requires the port projection

$$\Pi_{r,i} : \mathrm{Hol}_r(o) \longrightarrow \mathrm{Aut}^+(K_{r,i})$$

to be surjective. For every proper carrier automorphism $a \in \mathrm{Aut}^+(K_{r,i})$, A2 supplies a closed path γ_a and a projective unitary implementer $[U_a]$ satisfying

$$\Pi_{r,i}(\gamma_a) = a, \qquad \mathrm{Ad}_{[U_a]}\, D_{r,i}(v) = D_{r,i}(a\cdot v).$$

The implementer is endogenous to the same response:

$$[U_a] = [g_a c_a], \qquad g_a \in G^0_{D,r,i}, \qquad c_a \in C_{U(H_{r,i})}(\mathfrak{g}_{r,i}).$$

The centralizer factor acts trivially on $\mathfrak{g}_{r,i}$. A2 constrains operational meaning on accepted shared data and makes proper carrier rechartings inner on the complete port-response algebra. It does not imply global state extension, termination, confluence, unique normal forms, durable records, a global compact group, or a laboratory current [4].

A3 (maximum randomness under agreement constraints). In plain language, the selected state is least informative relative to the declared reference after every observer-visible constraint has been imposed. At finite regulator r, let $\mathcal{K}_r$ be the nonempty convex set of compatible local state families and let

$$\mathcal{D}_r(\rho\|\tau_r) = \sum_{P\in\mathcal{G}_r} w_{r,P} D(\rho_{r,P}\|\tau_{r,P}), \qquad w_{r,P} > 0.$$

The observer cover is state-determining on $\mathcal{K}_r$, and

$$\rho_r = \arg\min_{\rho\in\mathcal{K}_r} \mathcal{D}_r(\rho\|\tau_r).$$

A3 constrains state selection inside one A1-fixed feasible space. It does not select a field list, repair law, response map, particle multiplicity, or continuum limit [4].

None of the axioms contains a gauge group, a particle list, an event-manifold dimension, or a recovery law. A1 does fix a two-dimensional spherical support and hence the Lorentz kinematic type used below. The results below state which structures follow from these axioms, which require added premises, and which realization maps are consumed as premises.

1.3 The self-referential closure hypothesis

The closure hypothesis is the overarching structural premise. The universe is modeled as a timeless, closed, self-referential mathematical structure: the simulating description and the simulated system are one object, and the physical configuration is a fixed point $T(\mathfrak{U}) = \mathfrak{U}$ of a universe-level closure operator. No primitive or preferred global clock is assumed or derived. Repair order is a partial order on commits, and an observer history begins as an ordered chain of completed records. That order is invariant under arbitrary strictly increasing, including non-affine, regradings; affine time and proper time therefore require separate calibration and event-geometry receipts.

On the reading adopted here, a structure closed under its own description contains subsystems that read, record, compare, and reconstruct it. Within their emergent subjective time, observers assemble a consistent account of the structure, recover its simulator architecture, and construct that recovered specification. The constructed hardware and protocol generate the same class of observer records from which the specification was inferred. The universe thereby recognizes and realizes its own inner workings through its observer subsystems. Globally there is no earlier reconstruction and later construction; that ordering belongs only to the observers' internal records. The closure condition requires the recovered specification, its constructed realization, and the inhabited structure to return the same invariant quantities. It supplies no finite repair law, current derivative, physical kinetic operator, or existence and uniqueness proof for the cosmic fixed point.

No temporal paradox arises from this closure. The universe-level equation $T(\mathfrak{U}) = \mathfrak{U}$ relates parts of one timeless object; there is no global time coordinate along which information travels backward. An observer's subjective time is the ordered chain of completed records available to that observer. Repair descent is a separate well-founded order on authorized state updates, used to prove termination and compare admissible schedules. It is neither the observer's experienced time nor a universe-level clock. Observers can therefore recover the specification and later construct it in their own record order while both stages belong to the same global fixed point.

On this interpretation, A1–A3 provide finite consistency conditions under which the loop closes. A1 supplies a carrier with complete reversible response, A2 makes overlap transport endogenous to that response, and A3 selects a state without adding information beyond the agreement constraints. They are declared axioms of this realization and require no derivation from a prior temporal cause. The necessity claims proved in this paper are internal to the stated OPH premise sets. No uniqueness classification over every conceivable axiom system is asserted.

Two finite results give the hypothesis exact content on declared branches. The packet-quotient closure theorem proves, on the declared finite consensus branch, that the closure map is an affine idempotent whose fixed points are exactly the packets supported on consensus normal forms; its habitat-level extension is typed open [7]. The universe-level record-closure equation $N = \log M_0(\mathfrak{U}_N)$ is posed exactly. The all-rung arithmetic proves nonidentifiability for the bounded completion class defined by base agreement, positivity, and the carrier bound. The executable finite controls do not establish universal all-rung membership in a complete A1–A3 capacity-source contract, and no executable-to-Lean bridge supplies that step. Direct N is not evaluable on this incomplete source antecedent; the stronger source-class verdict is open [9]. A positive result must complete that antecedent and prove one physical zero, either within the three-axiom source contract or through a separately named stronger source law. The A2 endogeneity clause is the hypothesis's local expression within the axioms: every proper recharting is an operation implemented from inside the response.

The hypothesis also fixes how the declared carrier is to be read. Under closure there is no outside vantage from which an architecture could be tuned; the twelve-port carrier is part of the solution that observers inside the structure reverse engineer, and the axioms are the recovered description of that solution. This reading motivates the declaration and is typed as a hypothesis: the theorems of this paper consume the declared carrier as a premise and stand without the reading, and no uniqueness theorem over carriers is claimed. The quantitative closure candidates the hypothesis generates, the screen-grain equation and the capacity coordinate, are developed with full typing in Section 9.1. Existence, uniqueness, and stability of the physical fixed point are separate determinacy tests, and the finite theorems prove closure only on their declared finite branches.

1.4 How the constraints work together

Physical structure is selected at the intersections of constraints. The carrier incidence fixes a finite response space, its symmetry action, and its invariant subspaces. By itself, that geometry admits many equivariant response laws. A1 restricts the menu to faithful, complete, compact, commutator-closed reversible responses. A2 requires every proper recharting visible through overlap agreement to act internally on that same response. For the twelve-port carrier, the resulting one-dimensional fixed space excludes the sole centreless compact alternative and fixes the local Standard Model gauge algebra. Geometry, reversible dynamics, and agreement are all used in this conclusion.

Repair dynamics occupy a distinct part of the architecture. A named repair law produces a schedule-independent public record only when its termination and local-diamond premises hold. The simulator can test whether a proposed law realizes those premises and whether its records support the response and holonomy interfaces consumed by later branches. The gauge theorem does not silently import such a law: it consumes the complete response clause of A1 and the endogenous transport clause of A2 directly. Within OPH, the strange-loop architecture fixes the role of these clauses, and their theorem content is stated explicitly in A1 and A2. A concrete simulator implementation must separately verify that its response and transport data realize those axiomatic interfaces. This is an implementation-level realization check and adds no premise to the A1–A2 gauge implication.

The response metric uses a further conditional interface. A complete equally weighted census of centered port probes determines the finite kernels $QT^{2n}Q$. Slow-response normalization and the infinite-step limit select the low rank-three Gram ray. Quotient and metric completion apply after that limit. Completing at finite n gives a different object because the kernel has rank eleven on the centered space and rank six on the signed antipodal sector. The resulting completion reads carrier position only. Record order, repair cost, and exact load data remain a separate fiber.

The same rule applies to measures: carrier counting and algebraic traces supply finite candidate measures, agreement and repair determine whether they descend to the public quotient, and a physical realization map must then preserve them as spacetime or laboratory measures. This dependency discipline separates forced structure from formal resemblance.

1.5 The theorem chain

The results form branches with different premises:

$$
\begin{array}{rcl}
\text{finite carrier and convergent repair} & \longrightarrow & \text{consensus normal form,} \\
{\scriptstyle \text{signed port records} \atop \text{normalized repair response}} & \longrightarrow & {\scriptstyle \text{abstract continuous local} \atop \text{three-dimensional Euclidean carrier}}, \\
\text{algebra-state reading of the completed record} & \longrightarrow & \text{finite quantum record identities,} \\
\text{A1 response and A2 transport} & \longrightarrow & \text{Standard Model local Lie type,} \\
\text{displayed matter representation} & \longrightarrow & \text{conditional } \mathbb{Z}_6 \text{ matter image,} \\
\text{oriented spherical support} & \longrightarrow & \text{Lorentz observer-velocity kinematics,} \\
\text{realized null balance, Ward/Bianchi, and scale} & \Longrightarrow & G_{ab} + \Lambda g_{ab} = 8\pi G_N T_{ab}.
\end{array}
$$

The first five rows are finite or exact-limit results with the displayed premises. The sixth consumes the declared spherical support. The seventh is a reconstruction implication. A common modular–stress–entropy tower is the proposed route to its null-balance antecedent; its physical realization is an open realization map. Sections 2 through 9 develop these branches. Section 10 records the physical interpretation maps. Section 11 describes the verification stack. Section 12 states the falsification architecture and the empirical decision rule. Section 13 compares the program with neighboring approaches. The conclusion states the program's full ambition, a derivation of the constants and structural properties of nature from consistency requirements alone.

2 The primitive finite architecture

Definition 2.1 (Observer patch)**.** An observer patch is a finite object with local state, an observable boundary map, durable records, readback, and a set of authorized repair moves. A patch sees a fragment of the world: its own state and the boundary data of its authorized overlaps.

Definition 2.2 (Overlap and repair)**.** For patches x_i, x_j with an authorized overlap $e = (i, j)$, both induced boundary records must agree for the pair to hold public data. A repair move changes local state while preserving protected readout. A configuration is a normal form when no authorized repair applies.

The carrier realization used throughout is the twelve-port icosahedral architecture of A1: each carrier's boundary packet is combinatorially the oriented icosahedron boundary (P, E, F, o) with $|P| = 12$, $|E| = 30$, $|F| = 20$; carriers federate through typed seam algebras with coherent triple-overlap cocycles into a nerve carrying a degree-one bridge to the oriented spherical support. This is the declared source architecture. We do not claim a uniqueness theorem selecting it among all possible carriers; where results depend on it, the dependence is displayed.

Two exact features of this architecture enter the later constructions. First, the proper rotation group of the icosahedral packet is the alternating group A_5, with sixty rotations acting on the twelve ports; the real port-coefficient space has the character decomposition

$$P_{12} \;\cong_{A_5}\; \mathbf{1} \oplus \mathbf{3} \oplus \mathbf{3'} \oplus \mathbf{5}, \tag{1}$$

an exact multiplicity-free decomposition obtained by standard finite-group character theory [35] and checked by certified projectors. Second, record keeping is integer valued: a write appends $+1$, a retraction appends -1, and readback sums atomic events per port; a conservative repair transfers one unit across a seam whenever the oriented mismatch has magnitude at least two.

With $V(N) = \sum_i N_i^2$, each such repair strictly decreases V by $2(d-1)$ for mismatch $d \geq 2$, so repair terminates by a finite theorem. Divisibility of total load by twelve is a necessary consensus condition and is not sufficient by itself. For the certificate's source-generated full-pile packet, an explicit eighteen-move schedule reaches consensus.

2.1 Hardware and protocol: one substrate, typed dynamics, several readouts

The finite substrate and the operations acting on it are distinct parts of the model. Every physical conclusion requires a specified operation and its own realization map.

Under the closure hypothesis, the substrate is also the specification recovered by observers inside the model and constructed by them within their emergent subjective future. Its ports, seams, readback, records, and repair interfaces form the hardware side of the strange loop. The observations used to infer the specification and the constructed simulator that realizes it belong to the same globally closed structure.

The *substrate and reversible-response tangent* are the content of A1: a federation of finite carriers, each with the oriented twelve-port icosahedral boundary packet, joined through typed seam algebras with coherent triple overlaps into a nerve that bridges to the oriented spherical support, together with the complete reversible response space of each carrier. The companion microphysics paper specifies this substrate at the level of patch hardware, records, and synchronization interfaces [8]. This layer fixes what exists at each finite regulator: ports, seams, records, and response directions. It does not select a repair schedule, convergent repair law, or physical propagation operator.

The *agreement and state-selection protocol* combines A2 and A3 with named branch dynamics. Patches write and retract integer record events, read back port sums, compare induced records on shared boundaries, and interpret accepted data through the natural functor of A2. A named repair law moves oriented mismatches under quadratic descent; its termination and local-diamond premises are proved separately. A3 selects the residual state by constrained information projection after the agreement constraints are fixed. The companion consensus paper develops this layer as an asynchronous agreement protocol with explicit safety and liveness assumptions [7]. The protocol fixes what happens: which configurations count as repaired, which records become public, and which transports implement recharting.

The claim structure of the paper is that distinct readouts of this one protocol-on-hardware system yield the distinct pillars of observed physics. Table 2 states the correspondence and names the section that proves each step.

Two features of this pipeline carry the unification claim. The branches consume one substrate with typed operations: the same completed records that define measurement feed the event geometry, and the same overlap transport that defines agreement feeds the gauge forcing. Every arrow in the table is typed as a finite theorem, a certified computation, or an open realization map. The universe claim of the program is the conjunction: a physical carrier that realizes the hardware and runs the protocol produces public records whose readouts include quantum probability, Lorentzian causal structure, Einstein dynamics, and the Standard Model gauge and matter skeleton, with the listed open realization maps as its premises.

3 Observable normal forms and finite consensus

A completed finite configuration x is a fixed point of the named repair-and-agreement map:

$$\mathcal{R}_{\text{cons}}(x) = x, \tag{2}$$

Table 2: The reconstruction pipeline: typed operation, finite substrate, resulting structure, and its location in this paper.

Operation or branch premise	**Finite substrate**	**Resulting structure**	**Where**
Repair to a fixed point under quadratic descent and the transactional diamond	carrier records and seams	schedule-independent public record; measurement as completed consensus	§3
Algebra-state reading of the completed record surface	protected record projectors	Born weights, Lüders conditioning, Tsirelson bound	§4
Overlap transport around closed paths	seam groupoid and triple-overlap cocycles	central defects, edge sectors, edge-center entropy	§5
Record-germ separation along refinement	federation nerve and spherical bridge	Lorentz kinematics; conditional Lorentzian event manifold	§6
Modular flow, entropy stationarity, null tomography on one tower	the same record tower	conditional Einstein field equation	§7
Endogenous recharting of the complete port response	twelve-port incidence and response space	forced gauge Lie type $\mathfrak{u}(1) \oplus \mathfrak{su}(2) \oplus \mathfrak{su}(3)$; conditional matter image	§8
Capacity accounting and closure readback	screen capacity and grain	exact quantitative closures and physical tests	§9
Homogeneous propagation on a selected physical carrier action	the twelve-port vertex orbit or source-seam edge orbit	separate linked quartic and sixth-order coefficient rays with one rotated angular-rank-six template	§9.6

where $\mathcal{R}_{\mathrm{cons}}$ is the composite finite dynamics. This map is distinct from the universe-level closure operator T of Section 1.3. Fixed-point existence, observable determination, and schedule independence are separate questions. The following framework isolates them [5].

3.1 Exact observable fibers

Definition 3.1 (Observable quotient system)**.** An observable quotient system is a tuple

$$\mathfrak{S} = (Q, C, \mathcal{B}, B),$$

where Q is a set of configurations, $C \subseteq Q$ is the consistent subset, $\mathcal{B}$ is a set of protected records, and $B : Q \to \mathcal{B}$ is the record map. For $b \in \mathcal{B}$, write $C_b = C \cap B^{-1}(b)$.

The fiber C_b is unrealizable, reconstructing, or ambiguous according as its cardinality is zero, one, or greater than one. This trichotomy avoids assigning a state to a record that has either no consistent extension or several observationally indistinguishable extensions.

Theorem 3.2 (Canonical observable normalizer)**.** *The following statements are equivalent:*

(i) $B|_C$ *is injective;*

(ii) there is a unique map $N : Q \to C \sqcup \{\perp\}$ *that preserves* B *on its* C*-valued outputs, fixes every point of* C*, is constant on each* B*-fiber, and returns* $\perp$ *exactly on fibers with no consistent extension;*

(iii) $B|_C : C \to B(C)$ *is a bijection.*

If $B(Q) \subseteq B(C)$*, the map is a total idempotent retraction* $Q \to C$*.*

Proof. Injectivity makes every nonempty C_b a singleton, which defines N and forces all its properties fiber by fiber. Conversely, if $c, c' \in C$ have $B(c) = B(c')$, the fixed-point and fiber clauses give $c = N(c) = N(c') = c'$. The third statement is the restriction of the first to the image $B(C)$. □

This normalizer is defined by observable fibers, independent of a chosen repair schedule. A repair relation implements it only when the relation preserves B, reaches the consistent set, and satisfies an appropriate liveness condition. Confluence from one source does not by itself prove determination across distinct sources with the same protected record.

For a separately declared finite scheduler, protected observations admit four nested first-hit layers: nonempty consistent fiber, positive reach from some active source, almost-sure reach from every active source, and a single first-hit endpoint class modulo the declared silent equivalence. Their complementary cuts partition the empty-fiber, inaccessible, liveness, and selection obstructions, the layers instantiate the behavior-cut interface of this section exactly under explicit support–rewrite laws, and an exact finite-state morphism transports the whole profile. The four cuts compare coordinatewise, a product preorder rather than a scalar score. The definitions, machine-checked theorems, native fixtures, and countermodels are in the observable-normal-forms component paper [5]; the statement is conditional on the declared scheduler and morphism packet and supplies neither hitting-time rates nor an implementation-refinement result.

3.2 Approximate outputs and refinement

Let (Q, d_Q) be finite, let $(\mathcal{B}, d_{\mathcal{B}})$ be metric, and let $C = \Phi^{-1}(0)$ be nonempty for a nonnegative consistency residual Φ. For $t, r \geq 0$, define

$$\eta_\Phi(t) = \max\{\mathrm{dist}_Q(x, C) : \Phi(x) \leq t\}, \tag{3}$$
$$\omega_B(r) = \max\{d_Q(c, c') : c, c' \in C,\ d_{\mathcal{B}}(Bc, Bc') \leq r\}. \tag{4}$$

The first modulus turns residual error into distance from consistency. The second measures how well protected observations distinguish consistent outputs.

Theorem 3.3 (Sharp two-output stability bound)**.** *Suppose B is L_B-Lipschitz. If*

$$\delta_x, \delta_y, \varepsilon \geq 0, \qquad \Phi(x) \leq \delta_x, \qquad \Phi(y) \leq \delta_y, \qquad d_{\mathcal{B}}(Bx, By) \leq \varepsilon,$$

then

$$\begin{aligned} d_Q(x, y) \leq{}& \eta_\Phi(\delta_x) + \eta_\Phi(\delta_y) \\ &+ \omega_B(\varepsilon + L_B[\eta_\Phi(\delta_x) + \eta_\Phi(\delta_y)])\,. \end{aligned} \tag{5}$$

At zero residual the ω_B term is exact, and the coefficient one on each of the two residual-distance terms is optimal in the class of finite metric systems.

Proof. Choose nearest consistent points c_x, c_y. The two residual moduli bound the outer legs $x \to c_x$ and $c_y \to y$. Lipschitz continuity bounds $d_{\mathcal{B}}(Bc_x, Bc_y)$ by the argument of ω_B in (5); the inverse-observation modulus bounds the middle leg. The triangle inequality gives the result. Exactness at zero residual follows from the definition of ω_B. To test the first residual coefficient, take the two-point metric $Q = \{c, x\}$ with $C = \{c\}$, $d_Q(c, x) = s$, constant B, $y = c$, $\Phi(c) = 0$, and $\Phi(x) = 1$ at $(\delta_x, \delta_y, \varepsilon) = (1, 0, 0)$. The bound reduces to $s \leq \eta_\Phi(1) = s$, so that coefficient cannot be smaller than one. Exchanging x and y proves the same statement for the second coefficient. □

Let (Q_n, d_n) be nonempty finite metric spaces. For $m \geq j \geq n$, let $\rho_{m,n} : Q_m \to Q_n$ be restriction maps satisfying $\rho_{n,n} = \mathrm{id}_{Q_n}$ and $\rho_{m,n} = \rho_{j,n}\rho_{m,j}$. Let $N_n : Q_n \to Q_n$ be total level normalizers. Set

$$a_j = \max_{q \in Q_{j+1}} d_j(\rho_{j+1,j}N_{j+1}q, N_j\rho_{j+1,j}q), \qquad K_{j,n} = \mathrm{Lip}(\rho_{j,n}).$$

Theorem 3.4 (Accumulated refinement bound)**.** *For $m > n$ and $q \in Q_m$,*

$$d_n(\rho_{m,n}N_mq, N_n\rho_{m,n}q) \leq \sum_{j=n}^{m-1} K_{j,n}a_j.$$

In particular, if the restrictions are nonexpansive and $\sum_{j=0}^{\infty} a_j < \infty$, then

$$\sup_{m>n}\ \max_{q \in Q_m} d_n(\rho_{m,n}N_mq, N_n\rho_{m,n}q) \leq \sum_{j=n}^{\infty} a_j \longrightarrow 0 \quad (n \to \infty).$$

This is asymptotic naturality of the normalizers.

Proof. Insert the intermediate points $p_j = \rho_{j,n}N_j\rho_{m,j}q$. Consecutive points differ by at most $K_{j,n}a_j$. Summing the telescoping chain from p_m to p_n gives the bound. □

3.3 Computational boundary

The normalizer can be canonical without being cheap. For a Boolean circuit $F : \{0,1\}^n \to \{0,1\}$, take $C = F^{-1}(1)$ and let B_S project onto a declared coordinate set S.

Theorem 3.5 (Complexity of succinct boundary reconstruction)**.** *For succinct Boolean boundary systems:*

(i) deciding whether a protected record has a consistent extension is NP-complete;

(ii) deciding whether $B_S|_C$ is injective is coNP-complete, even when all but one variable are observed;

(iii) deciding whether every protected collar value has some consistent interior extension, equivalently whether an unrestricted total strong repair exists, is Π_2^P-complete.

Proof. For (i), a consistent extension is a polynomial witness and satisfiability is the case $S = \varnothing$. For (ii), noninjectivity is witnessed by two distinct satisfying assignments with the same projection; unsatisfiability reduces to injectivity after adding one hidden tie-breaking bit and one always-consistent baseline assignment. For (iii), a repair exists exactly when the corresponding circuit relation satisfies $\forall d\, \exists w\, R(w, d) = 1$, the canonical one-alternation quantified Boolean formula problem. ☐

3.4 Repair realization on the twelve-port carrier

For a prepared batch of support-closed transactions, join τ and σ in the conflict graph when

$$W_\tau \cap (R_\sigma \cup W_\sigma) \neq \varnothing \quad \text{or} \quad W_\sigma \cap (R_\tau \cup W_\tau) \neq \varnothing.$$

The conflict components below are obtained by iterating the following finite closure: compute semantic support to a fixed point inside every aggregate, rebuild the conflict graph on the expanded read and write sets, merge every connected component, and repeat until the partition and all aggregate supports are unchanged.

Proposition 3.6 (Transactional local diamond)**.** *Let an accepted repair be an aggregate transaction with read set R_τ, write set W_τ, a read snapshot, and a snapshot-determined payload. Assume:*

(i) every read set contains the semantic dependency closure of its write set, and every affected acceptance functional is revalidated at commit;

(ii) the terminal closure above exists, and each of its connected conflict components has one coherent canonical aggregate payload;

(iii) a prepared component whose snapshot remains valid survives commits of independent components; and

(iv) accepted commits preserve the protected record and strictly decrease the integer descent functional.

Then every one-step quotient peak

$$t \longleftarrow s \longrightarrow u$$

has a quotient state v with $t \longrightarrow v \longleftarrow u$.

Proof. Two different first steps cannot originate in one conflict component because that component has a unique aggregate payload. They therefore originate in distinct components of the terminal conflict closure. Rebuilding the graph after every aggregate support expansion makes their final read and write sets satisfy

$$W_\tau \cap (R_\sigma \cup W_\sigma) = \varnothing, \qquad W_\sigma \cap (R_\tau \cup W_\tau) = \varnothing.$$

Their writes are disjoint and neither commit changes the other's read snapshot. Semantic dependency closure ensures that neither commit changes an acceptance functional consumed by the other. Revalidation and the surviving component rule therefore make both second commits legal. Since the payloads are snapshot-determined and the writes are disjoint,

$$\mathrm{Apply}_\sigma \mathrm{Apply}_\tau(s) = \mathrm{Apply}_\tau \mathrm{Apply}_\sigma(s) = v.$$

Protected-record preservation makes this equality valid on the physical quotient. □

Theorem 3.7 (Consensus normal form)**.** *Suppose the accepted repair relation satisfies Proposition 3.6, the exact quadratic descent of Section 2, and repair completeness, meaning that its normal forms are exactly the consistent states. Every maximal asynchronous repair schedule from one initial state then terminates at the same consistent quotient normal form. The induced global repair map is idempotent, fixes consistent states, and preserves the protected record.*

Proof. Quadratic descent makes the accepted relation terminating. Proposition 3.6 makes it locally confluent, so Newman's lemma [28] makes it confluent. A terminating confluent relation has a unique normal form below each source. Repair completeness puts that normal form in the consistent set. Applying the normalizer a second time does nothing, and protected-record preservation holds along every accepted edge. □

Boundary 3.8. Atomic commits, disjoint write sets, or termination alone do not imply the local diamond. Countermodels omitting semantic dependency closure, canonical aggregation, or the surviving component rule are retained with the finite receipt. The reference engine exhaustively checks its states, peaks, payload hashes, protected records, and descent comparisons. The theorem is finite and asserts no continuum limit.

The declared class also has gauge compatibility (repair commutes with local relabeling), idempotence of the completed dynamics, and refinement compatibility where proved. The public object is the protected quotient record, not an arbitrary intermediate configuration.

The architecture sits close to distributed agreement in the tradition of Lamport, Shostak, and Pease [26]: patches play the role of protocol nodes, overlap repair of a quorum vote, and the completed fixed point of the decided state. A component paper of the stack proves safety and liveness for a Byzantine-tolerant reading of repair under explicit quorum and partial-synchrony assumptions [7]; the impossibility boundary of Fischer, Lynch, and Paterson [27] is the reason those assumptions are named.

4 Finite record algebra and quantum identities

Once a compare, write, and verify slice has completed, its accessible events are represented in a finite-dimensional $*$-algebra with a normalized state. This algebra-state representation is an explicit input at this stage. The consensus theorem identifies which record is public; it does not derive the classification of finite-dimensional C^*-algebras or the trace pairing.

Theorem 4.1 (Finite public-record identities)**.** *Let P_E be the projector of a completed public event E and ρ the normalized state of the completed record surface. The event weight and nonzero-weight update*

$$\Pr(E) = \operatorname{Tr}(\rho P_E), \qquad \rho \,|\, E = \frac{P_E \,\rho\, P_E}{\operatorname{Tr}(\rho P_E)} \tag{6}$$

define a normalized probability measure on every orthogonal event partition and a normalized positive state on the post-event corner. For self-adjoint dichotomic observables A_0, A_1 and B_0, B_1 in commuting record subalgebras, with $A_i^2 = B_j^2 = I$, define

$$S_{\text{CHSH}} = \operatorname{Tr}[\rho\{A_0(B_0 + B_1) + A_1(B_0 - B_1)\}]\,.$$

Then

$$|S_{\text{CHSH}}| \le 2\sqrt{2}. \tag{7}$$

Proof. Positivity of ρ and $0 \le P_E \le I$ give $0 \le \operatorname{Tr}(\rho P_E) \le 1$. For a complete orthogonal partition (P_k), linearity and $\sum_k P_k = I$ give $\sum_k \operatorname{Tr}(\rho P_k) = 1$. If the event weight is nonzero, $P_E \rho P_E / \operatorname{Tr}(\rho P_E)$ is positive, normalized, and supported on $P_E \mathcal{A} P_E$.

For $\mathcal{B} = A_0(B_0 + B_1) + A_1(B_0 - B_1)$, the commuting-subalgebra calculation gives

$$\mathcal{B}^2 = 4I - [A_0, A_1][B_0, B_1].$$

The two commutator norms are at most two, so $\|\mathcal{B}^2\| \le 8$ and $\|\mathcal{B}\| \le 2\sqrt{2}$. Taking its expectation in ρ proves the stated bound. □

Proposition 4.2 (Finite icosahedral CHSH candidate)**.** *On the declared binary-icosahedral spinor branch, the defining two-dimensional representation has a unique invariant line in its tensor square. For the incidence-defined (120)-row setting family carried by the twelve ports, the invariant singlet state gives*

$$|S_{\text{CHSH}}| = 1 + \frac{3}{\sqrt{5}} = 2.3416407865\ldots > 2. \tag{8}$$

Proof. The finite-group character sum gives invariant multiplicity one. Direct Pauli covariance under the (120) binary-icosahedral lifts identifies the singlet line. Every declared setting row has correlations $(-1/\sqrt{5}, -1/\sqrt{5}, -1/\sqrt{5}, +1)$, which gives the displayed value. An independent exact verifier reconstructs the invariant multiplicity, all (720) covariance identities, the joint probabilities, the no-signalling marginals, and the complete 12^4 setting census. □

The complete setting census contains (960) maximizers, including (480) with four distinct ports. The displayed (120)-row family forms two proper rotation orbits and is not uniquely selected by the carrier. The finite source packet supplies neither a completed two-wing record instrument nor source-selected settings. Proposition 4.2 is an exact projective candidate rather than a physical Bell prediction.

The first identity is the Born rule on this finite surface [18]; the second is Lüders conditioning [19]; the third is the Tsirelson bound [21] for the Clauser–Horne–Shimony–Holt combination [20] at fixed cutoff. Compatibility structure for commuting events follows the standard finite operational pattern within the same library.

Theorem 4.3 (Two-level partition expectation)**.** *Let $(P_i)_{i=1}^k$ be pairwise orthogonal projectors with $\sum_i P_i = I$. Define*

$$\mathcal{P}(X) = \sum_i P_i X P_i, \qquad \mathcal{A}(X) = \sum_i \frac{\operatorname{Tr}(X P_i)}{\operatorname{Tr}(P_i)} P_i,$$

with a zero projector contributing zero. Then $\mathcal{P}$ and $\mathcal{A}$ are positive, unital, trace-preserving idempotent linear maps. The exact range of $\mathcal{P}$ is the commutant of the partition; the exact range of $\mathcal{A}$ is the commutative span of the P_i; and

$$\mathcal{A}\mathcal{P} = \mathcal{A} = \mathcal{P}\mathcal{A}.$$

The average preserves every partition-event weight. For a partition member of nonzero weight, averaging commutes with Lüders conditioning and both orders give the normalized projector.

Proof. Orthogonality gives $\mathcal{P}^2 = \mathcal{P}$ and $\mathcal{A}^2 = \mathcal{A}$. Each map is a sum of positive corner maps, is unital, and preserves the trace. An operator is fixed by $\mathcal{P}$ exactly when its off-diagonal partition blocks vanish, which is equivalent to commuting with every P_i. An operator is fixed by $\mathcal{A}$ exactly when it is a scalar multiple of P_i on each nonzero block. These range descriptions imply $\mathcal{A}\mathcal{P} = \mathcal{A} = \mathcal{P}\mathcal{A}$. Finally, $\mathrm{Tr}(\mathcal{A}(X)P_i) = \mathrm{Tr}(XP_i)$; applying this identity to P_iXP_i and normalizing gives the conditioning statement. □

These are standard finite-matrix identities related to trace-preserving conditional expectations [22, 23]. The accompanying formal development contributes a checked arbitrary-partition interface, exact range and uniqueness statements, and adapters from projection events to the state-level Clauser–Horne–Shimony–Holt bound [6]. No priority claim is made for Born probability, Lüders conditioning, pinching, or Tsirelson's inequality as identities of the represented state.

At the level of weights, however, the Born form is derived rather than assumed. Every valuation on the effects of the finite surface that is nonnegative, normalized, and additive on coexisting effects equals $E \mapsto \mathrm{Tr}(\rho E)$ for a unique density operator ρ, in every finite dimension including two, where the projector-only Gleason theorem fails; the derivation consumes no continuity axiom. Finite webs do not by themselves supply its hypothesis. Sharp two-outcome webs admit the non-Born valuation $F_z(\mathbf{n}) = (1 + n_z^3)/2$. The exhibited unsharp trine excludes that particular response, but every axis in the finite battery has $n_y = 0$, so the distinct planar response $F_y(\mathbf{n}) = (1 + n_y^3)/2$ agrees with the maximally mixed Born weight on the whole battery while remaining non-affine off it. The source-attached real S_3 algebraic web obtained through the declared representation is likewise blind to the Pauli-Y direction. Its missing algebraic coordinate is exact: for its noncommuting projector candidates P, Q, the commutator phase lift $I/2 - (2\sqrt{3}/3)i(QP - PQ)$ is the $+Y$ projector, and $P, Q, +Y$ separate every fixed-trace 2×2 matrix. A generous closure under real coarse graining and real Kraus pullbacks remains Y-blind, so the complex phase operation is load-bearing. These finite controls locate, rather than erase, the missing step: a source-produced complex-tomographically-complete effect/instrument web and an operational theorem deriving full coexistent-effect additivity [6]. Outcome frequencies can validate such an instrument; they cannot create the universal valuation law by themselves. The algebraic phase lift is not a source instrument or an outcome receipt.

The measurement identities and the rigid dynamics of Section 10.5 share one state object: a composed frame-duality theorem proves that the unique Busch–Gleason state of the Heisenberg-shifted registered frame is exactly the propagator conjugation of the unique state, at every parameter of a supplied pointwise-continuous flow. The Schrödinger picture of the Born frame is forced by the Heisenberg action of the rigid generator; the flow itself stays a displayed premise and its parameter is not physical time. On the same registered interface, the Tsirelson bound and the no-signalling receipts are composed under one declared slot split rather than supplied eventwise: cross-party commutation is derived from the split, and local operations on one slot preserve every remote marginal through the same trace pairing.

Boundary 4.4. This is a finite operational record interface. It does not reconstruct an arbitrary quantum state space or an interacting continuum quantum field theory. Its role is narrower: the declared algebra-state representation of the completed consensus record carries the finite probability, conditioning, expectation, and correlation operations consumed by later branches.

5 Overlap defects and edge-center entropy

Overlap agreement has a second layer beyond equality of scalar records. Transport maps can compose only up to a central multiplier. The resulting defect is a finite gluing invariant; removing triangle defects and obtaining endpoint-only transport are distinct operations.

Theorem 5.1 (Central defect strictification and residual holonomy)**.** *Let N_Σ be the nerve of a finite overlap cover. Suppose $U_{ji} = U_{ij}^{-1}$ and*

$$U_{ij}U_{jk} = z_{ijk}U_{ik}, \qquad z_{ijk} \in Z_\Sigma,$$

where the identified coefficient group Z_Σ is abelian and overlap transport acts trivially on it. Then:

(i) z is a Čech 2-cocycle,

$$z_{jkl}z_{ikl}^{-1}z_{ijl}z_{ijk}^{-1} = 1;$$

(ii) its class $[z] \in \check{H}^2(N_\Sigma, Z_\Sigma)$ is invariant under changes of local frame;

(iii) a central edge rephasing removes every triangle multiplier exactly when $[z] = 0$;

(iv) after such a strictification, endpoint-only transport on a charge block $\mathcal{C}_\alpha$ holds exactly when the represented holonomy of every closed loop is the identity on $\mathcal{C}_\alpha$.

Proof. Associativity compares $(U_{ij}U_{jk})U_{kl}$ with $U_{ij}(U_{jk}U_{kl})$ and gives the cocycle identity. A local frame change conjugates the multiplier, hence leaves a central element unchanged. An edge rephasing by a 1-cochain changes z by its Čech coboundary, proving the third statement. Once the edge maps form a strict 1-cocycle, the discrepancy between two paths with common endpoints is the holonomy of the closed loop obtained by composing one path with the inverse of the other. □

The theorem gives a precise finite meaning to gluing curvature. A vanishing triangle class does not imply trivial transport around noncontractible loops. If the center varies as a local coefficient system, the cocycle equation must use the transported, twisted Čech differential; that case lies outside the untwisted statement above.

5.1 Finite edge-center decomposition

Let a regulated collar be cut into left and right halves with common interface Σ. Suppose finite-dimensional Hilbert spaces $\widetilde{H}_L, \widetilde{H}_R$ carry diagonal unitary actions of a finite compact group G_Σ. Write

$$\widetilde{H}_L = \bigoplus_\alpha V_\alpha \otimes H_{L,\alpha}, \qquad \widetilde{H}_R = \bigoplus_\beta V_\beta^* \otimes H_{R,\beta}.$$

Here the V_α are pairwise inequivalent irreducible unitary representations of G_Σ and $d_\alpha = \dim V_\alpha$.

Theorem 5.2 (Invariant collar decomposition and one-sided edge entropy)**.** *The invariant collar space and its sector-preserving algebra have the exact forms*

$$H_C = (\widetilde{H}_L \otimes \widetilde{H}_R)^{G_\Sigma} \cong \bigoplus_\alpha H_{L,\alpha} \otimes H_{R,\alpha},$$

$$\mathcal{A}_C = \bigoplus_\alpha \mathcal{B}(H_{L,\alpha}) \otimes \mathcal{B}(H_{R,\alpha}), \qquad Z(\mathcal{A}_C) = \bigoplus_\alpha \mathbb{C}\, P^C_\alpha.$$

Let $\Omega_\alpha \in V_\alpha \otimes V_\alpha^*$ be the normalized invariant vector corresponding to $d_\alpha^{-1/2} I_{V_\alpha}$. A sector-preserving collar state has the form

$$\rho_C = \bigoplus_\alpha p_\alpha \rho_\alpha, \qquad \rho_\alpha \in \mathcal{B}(H_{L,\alpha} \otimes H_{R,\alpha}),$$

where $p_\alpha \geq 0$, $\sum_\alpha p_\alpha = 1$, and each block with $p_\alpha > 0$ satisfies $\rho_\alpha \geq 0$ and $\operatorname{Tr} \rho_\alpha = 1$. Embed it in $\widetilde{H}_L \otimes \widetilde{H}_R$ through Ω_α and trace out $\widetilde{H}_R$. If $\rho_{L,\alpha} = \operatorname{Tr}_{H_{R,\alpha}} \rho_\alpha$, the resulting left state is

$$\rho_L = \bigoplus_\alpha p_\alpha \frac{I_{V_\alpha}}{d_\alpha} \otimes \rho_{L,\alpha},$$

and its entropy splits as

$$S(\rho_L) = H(p) + \sum_\alpha p_\alpha S(\rho_{L,\alpha}) + \sum_\alpha p_\alpha \log d_\alpha.$$

The edge term is the expectation in ρ_L of the sector observable

$$Z_L = \sum_\alpha (\log d_\alpha) P^L_\alpha,$$

where P^L_α projects onto $V_\alpha \otimes H_{L,\alpha} \subset \widetilde{H}_L$.

Proof. The tensor product of the two representation decompositions contains $V_\alpha \otimes V_\beta^*$. Schur's lemma gives a one-dimensional invariant space when $\alpha = \beta$ and zero otherwise, yielding the invariant direct sum and the center of its sector-preserving algebra. The partial trace of $|\Omega_\alpha\rangle\langle\Omega_\alpha|$ over V_α^* is I_{V_α}/d_α. The displayed expression for ρ_L follows. Entropy of a block-diagonal state is the Shannon entropy of the block weights plus the mean block entropy. Tensor-product additivity contributes $\log d_\alpha$ from each maximally mixed representation factor, and $\operatorname{Tr}(\rho_L Z_L) = \sum_\alpha p_\alpha \log d_\alpha$. ☐

The theorem distinguishes the gauge-invariant collar state from its one-sided reduced state; the representation-dimension term belongs to the latter. Reading Z_L as an area observable requires a physical realization map. Exact quantum Markovity alone also does not identify an arbitrary state-dependent Markov decomposition with these preselected edge-center factors; alignment of the state with the collar decomposition is an additional premise. Related edge-mode and algebra-center decompositions occur in lattice gauge theory [45, 46].

6 From observer velocities to events and spacetime

6.1 Exact kinematics of the spherical support

The oriented conformal spherical support carries

$$\mathrm{Conf}^+(S^2) \;\cong\; \mathrm{PSL}(2,\mathbb{C}) \;\cong\; \mathrm{SO}^+(3,1), \qquad H^3 \;\cong\; \frac{\mathrm{SO}^+(3,1)}{\mathrm{SO}(3)}, \qquad \dim H^3 = 3. \tag{9}$$

These are classical isomorphisms of conformal geometry, consumed as exact mathematics [34]. They identify the three-dimensional hyperbolic homogeneous space of future unit timelike directions. Interpreting those directions as physical observer velocities requires realized events, clocks, and local frames.

Boundary 6.1. Observer-velocity geometry and populated event geometry are distinct objects, and the program keeps them distinct. A populated four-dimensional event base requires record separation, local charts, an open-image condition, affine transition data, a quadratic cone, and causal reachability; these are required constructions, not corollaries of (9). The velocity space H^3 and event localization are typed as separate constructions, and no inference passes from one to the other without a certificate.

Theorem 6.2 (Sufficient gluing criterion for a record-germ event manifold)**.** *Let D be the algebraic direct limit of a cofinal sequence of finite record-germ stages. Suppose the refinement maps make the common-refinement distance $d : D \times D \to [0, \infty)$ a well-defined pseudometric, independent of the chosen stage representatives. Quotient D by zero distance and take its metric completion; call the result X. Then X is a separable, second-countable metric space. Suppose the same cofinal sequence supplies:*

(E1) population of every point of X by realized record germs;

(E2) a covering family of bi-Lipschitz homeomorphisms $\phi_i : U_i \to V_i$ from open subsets $U_i \subset X$ onto open subsets $V_i \subset \mathbb{R}^4$;

(E3) $C^{1,1}$ affine overlap maps satisfying the triple-overlap cocycle;

(E4) $C^{1,1}$ tetrads and inverse tetrads on those charts;

(E5) compatibility on overlaps: the chart metrics induced by the tetrads are related by pullback under the transition maps;

(E6) a nondegenerate quadratic event form of inertia $(1, 3)$, with compatible orientation and time orientation on every overlap; and

(E7) local semantic causal reachability agreeing with the selected cone.

Then the event base is a time-oriented Lorentzian four-manifold of regularity $C^{1,1}$. The future-unit timelike vectors, equivalently observer velocities modulo spatial rotations, form an H^3 fiber over each event. Stable causality and global hyperbolicity are additional conditions.

Proof. The algebraic direct limit of countably many finite stages is countable and dense in its metric completion, so X is separable. Every separable metric space is second-countable and Hausdorff. The covering bi-Lipschitz charts and their open images give the local Euclidean topology, while the affine cocycle gives a consistent $C^{1,1}$ atlas. The tetrads define chartwise Lorentz metrics, and the pullback clause makes them one global $C^{1,1}$ metric. Inertia, orientation, time orientation, and cone-compatible reachability supply the asserted local causal structure. The unit future timelike vectors of each Lorentzian tangent space form $\mathrm{SO}^+(3, 1)/\mathrm{SO}(3) \cong H^3$. □

6.2 A measured finite Lorentzian signature

The simulation branch measures the event form. Repair dynamics on federated carriers emits semantic events with ancestry; a declared chart combines one ancestry-depth coordinate with three spectral coordinates; a quadratic event form is fitted on a declared training half of the event pairs and evaluated on the held-out half. The signature is therefore held out from the update rule and is classified as a certified finite computation. The cone margin is the minimum held-out signed score, with sign $+1$ for ancestry-comparable pairs and -1 for incomparable pairs. A negative margin means that at least one held-out pair lies on the wrong side of the fitted quadratic cone.

On the reported support-adjusted comparison path with (carrier count, observer count, support width) equal to $(16{,}384, 128, 96)$, $(65{,}536, 256, 96)$, and $(262{,}144, 512, 384)$, the held-out event form has inertia $(1, 3)$ at every rung, with cone margins

$$-5.6, \qquad -3.2, \qquad -1.4 \tag{10}$$

on rows that differ in support width and cross-edge density, so the shrinking magnitudes do not form a convergence sequence. At 262,144 carriers the same-size support-width-96 control has 312 cross-observer edges and inertia $(2, 2)$, while the support-width-384 row has 1,062 edges and inertia $(1, 3)$. In this finite comparison, changing the support and cross-read structure changes the fitted inertia. The comparison does not isolate a unique mechanism or establish a continuum limit [2].

A preregistered fresh-seed replication of the two laptop-scale rungs, with frozen decision rules, five declared replicates per arm, an ancestry-permutation null, and a matched-density support control, reproduces the $(1, 3)$ form at 65,536 carriers on five of five replicates at both eigenvalue thresholds and returns FAILED overall: the 16,384-carrier rung is seed-fragile (two of five), the declared margin-ratio band is missed on all five replicates, and the ancestry-permutation null reproduces the threshold verdict in fourteen of fifteen cells, so the fitted signature is not attributed to ancestry structure by the current estimator. The emergent-rung reading of the measurement is demoted accordingly; the replication receipts and the frozen rule live with the instrument record. The measured form at scale survives replication, and the attribution and small-rung stability questions are the open instrument work.

Boundary 6.3. The chart allocates four coordinates by declaration; the instrument tests signature and full rank on that chart and does not derive the number of spacetime dimensions. The negative cone margins, one Euclidean local fit in the finite source domain below, closed neighborhoods in place of open charts, and the absence of a certified refinement limit prevent promotion of any finite object in this section to a continuum Lorentzian manifold. These conditions are recorded machine-readably in the receipts; the continuum attachment is an open realization map.

6.3 A finite source satisfying part of the local-domain contract

One deterministic capture at 16,384 carriers supplies: a finite causal complex of 2,304 events with exact acyclicity and a strict time function; a certified four-column chart whose global held-out form has inertia $(1, 3)$; six closed observer-visibility neighborhoods with exact induced affine transitions, cocycles, orientation, and time orientation; an observer-visible seam complex of 8,662 carriers and 11,816 seams whose 38 triangles are all frustrated (no global sign assignment satisfies every seam) under the declared orientation-reversing transport, with lift-ambiguity rank 3,117 over $\mathbb{F}_2$; typed scalar, chiral, and gauge sections whose sign-twisted local derivative passes exact integer adjoint, kinetic, covariance, gluing, refinement, and boundary checks; and a signed-graph rank theorem giving a zero twisted kernel, hence an exactly positive dimensionless spectral gap for the declared local operator. The capture is a certified finite computation; its rank and positivity statements are finite theorems. One provenance graph binds the construction, an isolated rerun reproduces the canonical receipt byte for byte, and the verifier rejects any receipt that promotes this finite domain to a continuum spacetime, a physical clock, or a physical mass scale [2].

7 Einstein reconstruction

Theorem 7.1 (Null tomography and metric ambiguity)**.** *Let V be four-dimensional with Lorentz form η. Suppose a symmetric form X satisfies $X(k, k) = 0$ for every null vector k. Then $X = \phi\eta$*

for a scalar ϕ. Null-null values therefore determine a symmetric tensor modulo the metric line. In an η-orthonormal basis with $\eta = \mathrm{diag}(-1, 1, 1, 1)$, set $s = \sqrt{3}/3$ and take the nine null vectors

$$(1, \pm1, 0, 0), \quad (1, 0, \pm1, 0), \quad (1, 0, 0, \pm1),$$
$$(1, s, s, s), \quad (1, s, s, -s), \quad (1, s, -s, s).$$

On the η-trace-free representative, use coordinates

$$x = (X_{00}, X_{01}, X_{02}, X_{03}, X_{11}, X_{12}, X_{13}, X_{22}, X_{23}), \qquad X_{33} = X_{00} - X_{11} - X_{22}.$$

The corresponding nine-charge design reconstructs the quotient

$$\mathrm{Sym}^2(V^*)/\mathbb{R}\eta.$$

Its determinant is $8192/27$. With the supremum norms on coordinates and charges, its exact decoder obeys

$$\|x\|_\infty \le (2 + \sqrt{3})\|q\|_\infty.$$

Proof. Choose coordinates with $\eta = \mathrm{diag}(-1, 1, 1, 1)$ and write every future null ray as $(1, n)$ with $|n| = 1$. The identity

$$X_{00} + 2X_{0i}n_i + X_{ij}n_in_j = 0$$

holds on the unit sphere. Its odd part gives $X_{0i} = 0$. The degree-two spherical-harmonic part gives $X_{ij} = \phi\delta_{ij}$, and the constant part gives $X_{00} = -\phi$. Thus $X = \phi\eta$. The quotient therefore has dimension $10 - 1 = 9$. Substitution of the nine displayed algebraic directions into the stated trace-free coordinate basis gives determinant $8192/27$. The explicit decoder has maximum absolute row sum $2 + \sqrt{3}$, which gives the supremum-norm bound. These two constants are exact analytic calculations. The accompanying Lean development proves that the nine directions are null and formalizes the design map, an explicit left-inverse decoder, injectivity, and the metric-line ambiguity [1, 10]. □

The four-dimensional small-ball coefficients used in entanglement-equilibrium arguments [41] can be isolated from their physical interpretation.

With $G_N > 0$ and $\ell > 0$, scalars $\delta S_{\mathrm{bulk}}, \delta A, t, f$ satisfying

$$\delta S_{\mathrm{bulk}} = \frac{8\pi^2\ell^4}{15}\, t, \qquad \delta A = -\frac{4\pi\ell^4}{15}\, f, \qquad \delta S_{\mathrm{bulk}} + \frac{\delta A}{4G_N} = 0 \tag{11}$$

obey $f = 8\pi G_N t$ exactly: substitution cancels the common nonzero prefactor. The identity isolates the numerical coefficients from their physical reading and is checked by machine.

The tensor completion begins at an explicitly realized null-balance relation. This is the point at which the physical bridge enters the mathematical implication.

Theorem 7.2 (Tensor completion of a realized null balance)**.** *Let (M, g) be a connected, time-oriented, C^3 Lorentzian four-manifold with Einstein tensor $G^{(g)}_{ab}$. Let T_{ab} be a symmetric C^1 tensor satisfying the independent Ward premise $\nabla^a T_{ab} = 0$. Suppose a common realized tower supplies a constant $\kappa > 0$ such that, at every point and for every null vector k,*

$$G^{(g)}_{ab}k^ak^b = \kappa T_{ab}k^ak^b, \tag{12}$$

and suppose $G_N > 0$ with $\kappa = 8\pi G_N$. Then there is a constant Λ such that

$$G^{(g)}_{ab} + \Lambda g_{ab} = 8\pi G_N T_{ab} \tag{13}$$

on M. If a reference event p_0 and scalar Λ_0 satisfy

$$(G^{(g)}_{ab} - 8\pi G_N T_{ab})\big|_{p_0} = -\Lambda_0 g_{ab}\big|_{p_0},$$

then $\Lambda = \Lambda_0$.

Proof. At each point, Theorem 7.1 applied to $G^{(g)} - \kappa T$ gives a scalar ϕ with $G^{(g)}_{ab} - \kappa T_{ab} = \phi g_{ab}$. The contracted Bianchi identity $\nabla^a G^{(g)}_{ab} = 0$, the Ward premise, and metric compatibility give $\nabla_b \phi = 0$. Connectedness makes ϕ constant. Set $\Lambda = -\phi$ and use $\kappa = 8\pi G_N$. Evaluation at p_0 gives $\Lambda = \Lambda_0$ under the final calibration premise. □

For OPH to supply the null-balance premise (12), one common refinement tower must realize all of the following:

(G1) the event manifold of Theorem 6.2, with sufficient regularity for its Levi–Civita connection and contracted Bianchi identity;

(G2) geometrically normalized cap modular flow and half-sided modular inclusions whose directional charges define one symmetric tensor T_{ab};

(G3) the Ward identity for that tensor, independently of its reconstruction from directional charges;

(G4) the edge-center split of Theorem 5.2, the physical normalization $\delta\langle Z_L\rangle = \delta A/(4G_N)$, the modular first law, and generalized-entropy stationarity;

(G5) one family with $\ell_r > 0$, $\ell_r \to 0$, all named remainders $o(\ell_r^4)$, the continuum diamond-kernel and fixed-volume area formulas used in the small-ball identity (11), and enough local observer directions to establish (12); and

(G6) universal coupling of the stress and entropy branches, a vacuum reference, and a physical scale calibration establishing $\kappa = 8\pi G_N$.

The exact finite layer contains null tomography, the edge-entropy split, and the coefficient identity. The joint physical tower, its continuum formulas, Ward identity, area normalization, clock and stress identifications, and scale readings are the open realization maps of the composition. The machine-checked composition proves the tensor step from an explicit null-balance premise and separately checks the small-ball arithmetic; it does not construct the bridge between them. The finite signature measurement in Section 6.2 tests one geometric ingredient and does not replace these premises. A machine-checked shell law carries one Newtonian ingredient on the committed carrier: a spherically symmetric flux with constant shell charge falls off exactly as the inverse square, with the exponent equal to the carrier dimension minus one, and the dimension three supplied by the completion theorem rather than assumed; the flux premises are declared register rows, and the join to the composed Einstein branch is open.

8 Two gauge reconstructions and a finite matter image

8.1 Sector reconstruction

The overlap and edge-center branch gives a structural gauge result before a particular compact group is identified.

Theorem 8.1 (Bosonic sector reconstruction)**.** *Suppose a cofinal refinement tail carries zero-obstruction, trivial-holonomy bosonic edge sectors and compatible fully faithful pullback functors. Suppose their closure* Sect_∞ *is an essentially small, additive, idempotent-complete, semisimple rigid symmetric* C^**-tensor category with simple unit and finite-dimensional Hom spaces. Suppose it has a faithful, unitary, strong symmetric monoidal,* $*$*-preserving fiber functor*

$$\mathcal{F} : \mathsf{Sect}_\infty \longrightarrow \mathsf{Hilb}_{\mathrm{fd}}.$$

Then

$$G_{\mathrm{Tan}} = \mathrm{Aut}_\otimes^{u,*}(\mathcal{F})$$

is compact in the topology of pointwise operator convergence and $\mathsf{Sect}_\infty \simeq \mathrm{Rep}(G_{\mathrm{Tan}})$ *as symmetric* C^**-tensor categories. The reconstructed group is unique up to isomorphism for the specified category and fiber functor.*

Proof. Here $\mathrm{Aut}_\otimes^{u,*}(\mathcal{F})$ denotes the unitary monoidal natural $*$-automorphisms of $\mathcal{F}$. For every object X, such an automorphism has a component in the compact group $U(\mathcal{F}(X))$. Naturality, tensor compatibility, symmetry, and $*$-compatibility are closed equations, so this automorphism group is a closed subgroup of the product $\prod_X U(\mathcal{F}(X))$ over a small skeleton and is compact in the subspace topology. The Doplicher–Roberts/Tannaka reconstruction theorem then gives the equivalence [48, 49]. A second compact group compatible with the same fiber functor is identified with the same automorphism group. □

Theorem 8.1 classifies the group encoded by the realized sector data. A trivial sector category reconstructs a trivial group. It does not select the Standard Model group without a suitable sector witness, and it does not identify its group with the finite response group below.

8.2 The twelve-port response algebra

Let A be the icosahedral adjacency operator and J the antipodal involution on the twelve ports. Exact incidence gives

$$10J = A^3 - 4A^2 - 5A + 10I. \tag{14}$$

Put $\varphi = (1+\sqrt{5})/2$. Choose from each antipodal axis one of the certificate's exact coordinate vectors $u_i \in \mathbb{Q}(\sqrt{5})^3$, each with $\|u_i\|^2 = 2+\varphi$, and put $U = [u_1 \ \cdots \ u_6]$. For a real port field f, write

$$b_i = \frac{f_i + f_{Ji}}{2}, \qquad d_i = \frac{f_i - f_{Ji}}{2}, \qquad c = \frac{1}{6}\sum_i b_i, \qquad b^0 = b - c\mathbf{1}.$$

The even coordinates split as $\mathbf{1} \oplus \mathbf{5}$. The odd coordinates split as the rank-three frame channel detected by U and its Galois-conjugate companion detected by $\sigma(U)$, where $\sigma(\sqrt{5}) = -\sqrt{5}$, giving the source-module decomposition

$$P_{12} \cong_{A_5} \mathbf{1} \oplus \mathbf{5} \oplus \mathbf{3} \oplus \mathbf{3}'.$$

Theorem 8.2 (A1–A2 gauge Lie-type forcing)**.** *Let* $D : P_{12} \to \mathfrak{u}(H)$ *be the complete reversible response derivative supplied by A1, and put* $\mathfrak{g} = D(P_{12})$*. Suppose the endogenous transport clause of A2 covers every proper carrier action* $a \in A_5$ *and gives projective implementers satisfying*

$$\mathrm{Ad}_{[U_a]} D(v) = D(a\cdot v), \qquad [U_a] = [g_a c_a],$$

where $g_a \in G_D^0$ *and* c_a *centralizes* $\mathfrak{g}$ *pointwise. Then*

$$\mathfrak{g} \cong \mathfrak{u}(1) \oplus \mathfrak{su}(3) \oplus \mathfrak{su}(2).$$

The center is the unique trivial line. Up to exchanging $\mathbf{3}$ *and* $\mathbf{3}'$ *by the outer automorphism of* A_5*, the* $\mathfrak{su}(2)$ *ideal carries* $\mathbf{3}$ *and the* $\mathfrak{su}(3)$ *ideal carries* $\mathbf{3}' \oplus \mathbf{5}$*.*

Proof. Injectivity of D gives $\dim \mathfrak{g} = 12$. Its commutator-closed image is a finite-dimensional Lie subalgebra of $\mathfrak{u}(H)$, and the positive trace pairing makes it compact. For every $a \in A_5$, the centralizer factor c_a acts trivially on $\mathfrak{g}$, so $\mathrm{Ad}_{[U_a]}|_{\mathfrak{g}} = \mathrm{Ad}_{g_a}|_{\mathfrak{g}}$ is inner. The chosen projective lifts need not form a homomorphic section. Covariance through the injective map D nevertheless gives the homomorphism

$$a \longmapsto (D(v) \mapsto D(a\cdot v)) \quad \text{from } A_5 \text{ to } \mathrm{Int}(\mathfrak{g}).$$

The transitive proper-carrier action fixes only the uniform port line, hence $\dim \mathfrak{g}^{A_5} = 1$.

Compactness gives the reductive decomposition $\mathfrak{g} = \mathfrak{z} \oplus [\mathfrak{g}, \mathfrak{g}]$. Inner automorphisms fix the center pointwise, so $\dim \mathfrak{z} \leq 1$.

If $\mathfrak{z} = 0$, the low-dimensional classification of compact simple Lie algebras forces a twelve-dimensional compact semisimple algebra to be $\mathfrak{su}(2)^4$ [47]. Inner automorphisms preserve each simple ideal. On each three-dimensional ideal, the restriction of the A_5 action is either trivial or faithful because A_5 is simple. Its fixed-space dimension is therefore three or zero: a faithful A_5 subgroup of SO(3) cannot fix an axis, since a finite rotation group fixing an axis is cyclic. The fixed-space dimension of $\mathfrak{g}$ would be a multiple of three, contradicting the one-dimensional fixed line. Hence $\dim \mathfrak{z} = 1$.

The semisimple part has dimension eleven. The same low-dimensional classification leaves only $11 = 8 + 3$, realized by $\mathfrak{su}(3) \oplus \mathfrak{su}(2)$. Adding the center proves the Lie-algebra claim. The center must be the unique fixed line. The three-dimensional simple ideal cannot carry the trivial action, since that would add three fixed directions; hence it carries $\mathbf{3}$ or $\mathbf{3}'$, and the eight-dimensional ideal carries the complementary triplet together with $\mathbf{5}$. ☐

Boundary 8.3*.* No ambient compact group or candidate Lie algebra occurs among the premises of Theorem 8.2. A1 supplies the faithful compact commutator-closed response algebra, and A2 supplies its inner proper-carrier action. The theorem fixes the abstract local Lie-algebra type. The source receipts derive the carrier module, the inverse-port response, and its relative band signs. The source contains no reconstruction of D from ordered reversible histories or same-current holonomy; the identification consumes both as premises. The physical matrix current, couplings, matter action, global quotient, and laboratory attachment are the open realization maps of this branch. For the registered adjacency recurrence and inverse-port readback, the full response-word algebra is exactly $\mathrm{span}\{I, A, A^2, A^3\}$. It is four-dimensional and commutative. It cannot contain twelve independent current generators, a nonzero bracket, or any nonidentity proper recharting. This bounded obstruction does not range over order-sensitive port perturbations. A source-positive realization requires both composition orders on one carrier, twelve independent first-order port derivatives, exact bracket closure with derived rank eleven and the constant generator combination spanning the one-dimensional center, and closed words implementing all sixty proper rechartings. The target-free diagonal phase lift supplies rank twelve with an abelian bracket. Adjoining the registered connected adjacency tangent generates $\mathfrak{u}(12)$ and a derived algebra of rank 143. It is an exact negative control for that direct lift and identifies the need for a non-diagonal source response. It does not exclude other port actions. Conditional on the canonical oriented carrier, an exact Reynolds calculation gives

$$\dim_{\mathbb{Q}} \mathrm{Hom}_{A_5}(\Lambda^2\mathbb{Q}^{12}, \mathbb{Q}^{12}) = 14.$$

The serialized rational basis is the complete target-free alternating-bracket search space. It does not choose a bracket. The complete Jacobi condition has quadratic coefficient-row rank 38, with an exact $11 + 27$ rowspace decomposition. Conditional on three named compact-Lie inputs, the real compact locus is classified into P, F, G; its noncompact components, reconstruction from ordered source histories, and same-current holonomy remain separate gates [2, 1]. The port count twelve equals the dimension of the forced algebra, so A1 fixes the dimension by declaration; the content of the theorem is the exclusion, through compactness and the one-dimensional fixed space, of the only other twelve-dimensional compact type $\mathfrak{su}(2)^4$, and the forcing of the unique center. No uniqueness claim over carriers is made, and under the closure hypothesis of Section 1.3 the carrier is read as part of the reconstructed solution. This interpretation does not enter the theorem, whose declared premises stand independently.

Let

$$\Phi_0(b^0) = \sum_{i=1}^{6} b_i^0 u_i u_i^\mathsf{T}, \qquad \widehat{x}\, y = x \times y.$$

The conditional matrix witness uses the space

$$H = \mathbb{C}^3_E \oplus \mathbb{C}^3_W$$

and four nonzero rational coefficients $\lambda_1, \lambda_5, \lambda_3, \lambda_{3'}$, one on each source band. These are explicit branch premises. The two three-dimensional response blocks match the two rank-three odd channels; they are not asserted to be a derived physical Hilbert space. Write v_p for the signed coordinate vector at port p. For $g \in A_5$, let $R_g \in \mathrm{SO}(3)$ be the exact rotation satisfying $R_g v_p = v_{g(p)}$ on the oriented port frame, and define

$$\Pi(g) = \mathrm{diag}(R_g, \sigma(R_g)).$$

The target A_5 action on $\mathfrak{u}(H)$ is conjugation by $\Pi(g)$.

Theorem 8.4 (Conditional four-band finite port-response algebra)**.** *On the declared twelve-port carrier and supplied response representation, define*

$$\begin{aligned} K(f) = \mathrm{diag}\big(&\lambda_3 \widehat{Ud} + i\{\lambda_1 c I_3 + \lambda_5 \Phi_0(b^0)\},\\ &\lambda_{3'} \widehat{\sigma(U)d}\big) \in \mathfrak{u}(H). \end{aligned} \tag{15}$$

Then K is injective, its image is commutator-closed, and

$$K(g\cdot f) = \Pi(g) K(f) \Pi(g)^*.$$

The induced A_5 action on $\operatorname{im} K$ is inner. Let $K^{-1} : \operatorname{im} K \to P_{12}$ denote the inverse of K onto its image. With the bracket pulled back from that image,

$$[x, y]_K = K^{-1}([K(x), K(y)]_{\mathrm{mat}}),$$

one has

$$(P_{12}, [\,\cdot\,,\cdot\,]_K) \cong \mathfrak{u}(3) \oplus \mathfrak{so}(3) \cong \mathfrak{u}(1) \oplus \mathfrak{su}(3) \oplus \mathfrak{su}(2). \tag{16}$$

Its center is the uniform one-dimensional port line, its derived algebra has dimension eleven, and the five-dimensional A_5 band is noncentral. The map has a positive-definite invariant Hilbert–Schmidt pullback.

Proof. Equation (14) is obtained by evaluating both sides on the four distinct adjacency eigenspaces $\mathbf{1}, \mathbf{3}, \mathbf{3}', \mathbf{5}$. The source protocol solves the common farthest-shell filter on each port and gives the signed response $-J$.

The constant coordinate maps to $i\mathbb{R}I_3$. The six rank-one axis matrices $u_i u_i^\mathsf{T}$ form a basis of the symmetric 3×3 matrices and satisfy

$$\sum_i u_i u_i^\mathsf{T} = (5 + \sqrt{5})I_3.$$

Their sum-zero coefficients therefore map isomorphically to the traceless symmetric matrices under Φ_0. The first two even terms span $i\,\mathrm{Sym}_3(\mathbb{R})$. The map U is an isomorphism on the $\mathbf{3}$ odd band and annihilates the $\mathbf{3}'$ band; $\sigma(U)$ has the complementary property. Their hat maps span one copy of $\mathfrak{so}(3)$ in each response block. Because every $\lambda_\bullet$ is nonzero, these four images are independent and K has rank $1 + 5 + 3 + 3 = 12$.

In the first block, $i\,\mathrm{Sym}_3(\mathbb{R}) \oplus \mathfrak{so}(3) = \mathfrak{u}(3)$ as a real Lie algebra, while the second block is $\mathfrak{so}(3)$. Thus the image is exactly $\mathfrak{u}(3) \oplus \mathfrak{so}(3)$ and is closed under commutators. Pullback of the matrix commutator satisfies bilinearity, antisymmetry, and Jacobi and gives (16). The derived algebra is $\mathfrak{su}(3) \oplus \mathfrak{so}(3)$, of dimension $8 + 3 = 11$; the remaining line is the center of $\mathfrak{u}(3)$. Direct evaluation identifies it with the uniform port vector and exhibits a nonzero commutator involving the five-dimensional band. Equivariance and positive definiteness of the pairing $-\operatorname{Re}\operatorname{tr}(K(f)K(f'))$ are exact calculations in $\mathbb{Q}(\sqrt{5})$, verified over the full sixty-element domain by the exact-arithmetic certificate with its manifest, receipt, and negative controls [1]; by the exhaustive-computation rule of Table 1 these calculations are part of the proof. The same certificate verifies, for all sixty elements, that $\Pi(g)$ is the exponential of an element of $\operatorname{im} K$; hence its conjugation action is inner. The finite refinement and persistence diagrams recorded in the certificate are checked there as separate commuting-square computations. □

Boundary 8.5. The A_5-module decomposition alone does not determine a Lie bracket: the same module also carries the zero bracket, whose trivial response-generated group fails the A2 endogeneity clause. The independent inputs to Theorem 8.4 are the carrier incidence and orientation, the supplied response representation, and four nonzero band coefficients. The block architecture and coefficients are declared. No measured particle, mass, or coupling value enters the exact verification. The theorem constructs the pulled-back bracket for this response map; it does not select K from source histories or prove a uniqueness claim over all A_5-equivariant brackets or finite carriers. The abstract Lie type is forced independently by Theorem 8.2.

8.3 Global form and anomaly-free exterior module

Theorem 8.6 (Finite matter image and exhaustive exterior-menu selection)**.** *Let*

$$V = \mathbb{C}^3_{-1/3} \oplus \mathbb{C}^2_{1/2}$$

be an additional, explicitly supplied trace-balanced matter representation of the Lie algebra forced in Theorem 8.2. This $3 \oplus 2$ matter space is distinct from the supplied response space $H = \mathbb{C}^3_E \oplus \mathbb{C}^3_W$ and is not derived from it. Let

$$\widetilde{G} = \mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$$

act on V by

$$(g_3, g_2, z) \cdot (v_3, v_2) = (z^{-2} g_3 v_3, z^3 g_2 v_2).$$

The differential of this action gives the displayed hypercharges. Its exterior module has the exact branching

$$\Lambda^2 V \oplus \Lambda^4 V = Q \oplus u^c \oplus e^c \oplus d^c \oplus L \tag{17}$$

with

component	$\mathrm{SU}(3)\times\mathrm{SU}(2)$ *type*	Y
Q	$(3,2)$	$1/6$
u^c	$(\bar{3},1)$	$-2/3$
e^c	$(1,1)$	1
d^c	$(\bar{3},1)$	$1/3$
L	$(1,2)$	$-1/2$

This is the familiar one-generation $\mathbf{10}\oplus\overline{\mathbf{5}}$ *branching associated with* $\mathrm{SU}(5)$ *unification [50]. It has total complex dimension fifteen. All perturbative gauge and mixed gravitational anomalies vanish, and the number of weak doublets, counted with color multiplicity, is four. The common kernel of the action of* $\widetilde{G}$ *on* V*, and hence on* $\Lambda^2V\oplus\Lambda^4V$*, is* $\mathbb{Z}_6$*. Hence the maximal faithful matter image is*

$$S(U(3)\times U(2))\cong\frac{\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)}{\mathbb{Z}_6}. \tag{18}$$

Define

$$G_{\text{packet}}:=S(U(3)\times U(2)),$$

the maximal faithful image of the displayed declared exterior module. This quotient is familiar as a possible global form of the Standard Model gauge group [52]. The scan menu consists of the ten nontrivial irreducible summands of $\Lambda^\bullet V$ *after the invariant vacuum* Λ^0V *and invariant top line* Λ^5V *have been excluded by declaration. Among all* $2^{10}=1{,}024$ *subsets of this menu, imposing nonemptiness, chirality, and vanishing of the mixed gravitational–*$\mathrm{U}(1)$*,* $\mathrm{SU}(3)^2\mathrm{U}(1)$*,* $\mathrm{SU}(2)^2\mathrm{U}(1)$*, and* $\mathrm{U}(1)^3$ *traces leaves exactly two rank-fifteen masks. They are exchanged by charge conjugation, and the exterior-parity grading is computed as an output.*

Proof. The exterior-sum identity

$$\Lambda^k(A\oplus B)=\bigoplus_{p+q=k}\Lambda^pA\otimes\Lambda^qB$$

gives the displayed five components and charges. The anomaly-cancellation conditions are standard [51]. With the conventional quadratic indices $T(3)=T(2)=1/2$, the pure color anomaly is

$$\mathrm{SU}(3)^3:\qquad 2-1-1=0,$$

where the two color fundamentals occur in the weak doublet and the two antifundamentals are u^c and d^c. The local $\mathrm{SU}(2)^3$ anomaly vanishes identically because the doublet is pseudoreal. The mixed anomalies are

$$\mathrm{SU}(3)^2\mathrm{U}(1):\quad \frac{1}{6}-\frac{1}{3}+\frac{1}{6}=0,\qquad \mathrm{SU}(2)^2\mathrm{U}(1):\quad \frac{1}{4}-\frac{1}{4}=0.$$

The abelian cubic and mixed gravitational sums are

$$6\Big(\frac{1}{6}\Big)^3+3\Big(-\frac{2}{3}\Big)^3+1+3\Big(\frac{1}{3}\Big)^3+2\Big(-\frac{1}{2}\Big)^3=0,$$

$$6\Big(\frac{1}{6}\Big)+3\Big(-\frac{2}{3}\Big)+1+3\Big(\frac{1}{3}\Big)+2\Big(-\frac{1}{2}\Big)=0.$$

There are three color copies of the Q doublet and one L doublet, so the Witten parity is even [53].

For the displayed cover action, the element

$$\left(e^{2\pi i/3}I_3, -I_2, e^{i\pi/3}\right)$$

acts trivially on every row of the table and generates a cyclic group of order six. Direct center-action enumeration proves that no larger common kernel exists; the independent integer presentation has Smith invariants $(1, 1, 1, 1, 1, 6)$. The cover action has determinant one on V, so $\Lambda^4 V \cong V^*$ equivariantly. Its kernel on the displayed exterior module therefore equals its kernel on V. This gives (18). Finally, the finite scan evaluates nonemptiness, chirality, and the four displayed scan traces on all 1,024 masks in the declared ten-summand menu. Its only survivors are the two exterior-parity masks; the explicit conjugation permutation exchanges them. Parity is recorded after selection and is not a selection predicate. □

The structure layer composes as one conditional statement with a coarse two-mask capstone. A single theorem on the registered premise bundle proves the forced Lie type, the unique type-matched nearest family at the bundle's balanced point, and the fifteen-state count, anomaly forms, and $\mathbb{Z}_6$ stabilizer directly on the bundled selection mask. Within the declared exterior-table grammar, exactly two masks survive at the selected coarse G label and the mask permutation exchanges them. This does not count bracket points in the continuous G family or select a global form, physical matter carrier, current, or action, so it is not a two-model uniqueness theorem for complete Standard Model structures.

Boundary 8.7. The finite response group G_{packet} and the sector-reconstructed group G_{Tan} arise from different data. Their equality is a separate commuting-square premise. Laboratory current attachment, physical matter identification, exclusion of extra light sectors, scalar attachment, family multiplicity, Yukawa couplings, mixing, masses, and a continuum quantum-field realization are the open realization maps of the matter branch. The theorem classifies a finite declared exterior representation and its maximal faithful image. The exclusion of $\Lambda^0 V$, $\Lambda^5 V$, direct sums, vectorlike additions, and neutral singlets is not derived by this scan. A term-by-term correspondence table classifies every term of the textbook Standard Model Lagrangian by what the reconstruction supplies for it: an exact conditional theorem, a declared premise, partial structure stated exactly, or nothing, with the premises and evidence on each row. The table states how much of the Lagrangian the reconstruction supplies and names the open realization maps for the rest.

8.4 Massless carriers, conditionally

Three conditional masslessness statements occur on distinct premise sets. They are consistency consequences of their declared branches and supply no independent discriminator from the Standard Model or General Relativity.

The photon. A finite one-loop calculation imports the Standard Model electroweak action and its symmetry-restored vacuum relation. The calculation certifies a transverse photon self-energy and a null direction of the neutral mass matrix, hence no hard photon mass term on that fixture. This validates the calculation harness. An OPH derivation additionally requires a source-generated physical action, laboratory-current attachment, and photon-pole map.

The gluon. The color factor in (16) lies in the unbroken subalgebra of the conditional global form. Under the separately declared scalar completion, no scalar direction carries color and the finite action contains no hard color-vector mass term. This statement does not construct the physical gluon spectrum. Confinement and hadron physics require strong-coupling dynamics outside the finite calculation.

The graviton. The Einstein branch of Theorem 7.2 carries a classical spin-two carrier exactly as far as the composition carries the field equation. Linearizing the composed relation about a suitable vacuum yields a null spin-two kernel with two transverse-traceless modes, conditional on the same premise vector. The construction supplies no source-selected physical Hilbert space, positive-residue graviton pole, or graviton rest-mass prediction.

The photon and gluon statements concern finite actions with declared inputs. The graviton statement inherits every premise of the Einstein composition.

9 Exact quantitative closures and physical tests

The finite architecture supports a small set of quantitative surfaces sharp enough to be tested numerically. Each surface below is stated with its type from Table 1: the theorems are finite theorems, the numerical enclosures are certified finite computations, and every empirical surface is typed as a diagnostic, a registered conditional test, or a frozen prospective branch prediction. The closure hypothesis of Section 1.3 contributes its quantitative candidates here, in Section 9.1, under the same typing.

9.1 Closure candidates: the screen grain and the capacity coordinate

In the displayed proposals, $A_T(\cdot)$ is the inverse electromagnetic coupling the declared Thomson-limit source map returns at a trial screen grain, $\alpha_U(\cdot)$ the coupling the same declared map returns at its unification point, $\mathfrak{U}_N$ the regulated universe-level observer system at capacity budget N, and M_0 the correctable-code capacity of its terminal public record; the grain scalar of these equations is distinct from the port set and the event projectors used elsewhere in this paper. Certified roots of both displayed maps are compared against the 2022 CODATA inverse coupling [59]. The symbol $\varphi = (1 + \sqrt{5})/2$ is the golden ratio of Section 8.

The closure principle. The axioms describe the observer screen. The global closure principle states that the simulating and the simulated description are one system. Every quantity that has both a construction-side reading and a readback-side reading must take the same value once a typed bridge proves that the two readings denote one invariant. The equality is forced by self-identity. Constructing that bridge and the return map remains part of the physics. Existence, uniqueness, and stability are separate determinacy tests on the resulting equation. The name given to such an equation carries no mathematical weight: after the typed identification, unequal readings would describe two systems rather than the single self-referential universe.

The local screen-grain proposal

$$P = \varphi + \sqrt{\pi}/A_T(P),$$

with $A_T(P)$ the inverse coupling returned by the declared source map, has one interval-certified fixed point on its declared analytic domain. Interpreting that root as the laboratory coupling additionally requires a target-independent choice of the physical map, a typed bridge identifying the two sides as readings of one quantity, and same-scheme hadronic spectral transport to the Thomson endpoint. The displayed map's certified fixed point sits at $P = 1.6309720959$ and returns $A_T = 136.9948352$; the certified gauge-width variant's fixed point sits at $P = 1.6309682414$ and returns 137.0356601. Through the same outer equation the measured 2022 CODATA coupling $\alpha^{-1} = 137.035999177(21)$ reads back the grain value $P = 1.6309682094$. The comparisons stand at 3.0×10^{-4} relative for the displayed map and at 2.5×10^{-6} for the gauge-width variant, as diagnostics of the declared source maps.

The direct global proposal is

$$N = \log M_0(\mathfrak{U}_N).$$

The bounded completion class defined by base agreement, positivity, and the carrier bound does not select a unique cosmic value. Universal all-rung membership in a complete A1–A3 capacity-source contract and the corresponding executable-to-Lean bridge are absent. Direct N is not evaluable on that incomplete antecedent, and the stronger source-class verdict is open. A positive result must complete the source antecedent and prove one physical zero, either within the three-axiom source contract or through a separately named stronger source law. A separate common-load branch starts from

$$N_0 = \pi \exp\!\left(\frac{6\pi}{P\,\alpha_U(P)}\right).$$

On the finite collar branch, the declared total reserve expectation $P/4$ and six-class equidistribution give presence probability $P/24$ for each declared class. If one class is physically selected as the blocked event, its scalar-weighted presence receipt is discharged, and that normalized collar-survival factor is proved to act on the global capacity, the corresponding candidate is

$$N_{\rm pres} = N_0\left(1 - \frac{P}{24}\right), \qquad \ln\frac{N_{\rm pres}}{\pi} = \frac{6\pi}{P\,\alpha_U(P)} + \ln\left(1 - \frac{P}{24}\right).$$

The exponential alternative $N_{\rm Pois} = N_0 e^{-P/24}$ requires a separate mean-count or continuum carrier. Exact neutral, one-class, and six-class-total actions share the declared local datum, obey positive composition and finite cut-count regrouping, and disagree globally. The finite source therefore selects no action or blocked-event semantics. The named-law branch is not evaluable on this source class, and its horizon branch has no capacity object. A stronger source-derived action would require the screen/electroweak bridge, physical common-load bridge, physical $\mathbb{Z}_6$ seam action, scalar-weighted receipt, and horizon-record identification. On the source-forward numerical branch,

$$N_0 = 3.5321 \times 10^{122},$$

so the finite-presence candidate is 3.2921×10^{122} and the exponential candidate is 3.3001×10^{122}. The full weighted Planck base-ΛCDM chain gives the comparison coordinate 3.3129×10^{122}. The respective residuals are -0.63 percent and -0.39 percent. Both comparisons were exposed before the branch choice and carry no predictive weight.

9.2 The positive-chamber Koide identity

The icosahedral carrier's face structure supplies order-three cyclic response operators. The following exact identity governs every Hermitian three-cycle response [13].

Theorem 9.1 (Positive-chamber Koide identity). *Let $R^3 = I$ generate a cyclic action and let*

$$C = aI + bR + \bar{b}R^2, \qquad a > 0, \quad b = \rho e^{i\delta} \in \mathbb{C},$$

be the associated Hermitian circulant, with eigenvalues

$$\lambda_k = a + 2\rho\cos\!\left(\delta + \frac{2\pi k}{3}\right), \qquad k = 0, 1, 2.$$

Suppose all three eigenvalues are nonnegative and, for one scale $s > 0$, $\sqrt{m_k} = \sqrt{s}\,\lambda_k$. Then

$$Q := \frac{\sum_k m_k}{\left(\sum_k \sqrt{m_k}\right)^2} = \frac{1}{3} + \frac{2}{3}\left(\frac{\rho}{a}\right)^2, \qquad \textit{so} \qquad Q = \frac{2}{3} \iff \frac{\rho}{a} = \frac{1}{\sqrt{2}}. \tag{19}$$

Proof. The three cosines at angles separated by $2\pi/3$ sum to zero, and their squares sum to $3/2$. Hence $\sum_k \lambda_k = 3a$ and $\sum_k \lambda_k^2 = 3a^2 + 6\rho^2$. With $m_k = s\lambda_k^2$ and $\sqrt{m_k} = \sqrt{s}\,\lambda_k$, the ratio is

$$Q = \frac{s(3a^2 + 6\rho^2)}{s(3a)^2} = \frac{1}{3} + \frac{2}{3}\left(\frac{\rho}{a}\right)^2 .$$

The equivalence follows by solving $Q = 2/3$ for ρ/a on $a > 0$, $\rho \geq 0$. □

The phase δ cancels from Q and jointly controls the two mass ratios, so the identity constrains one combination of the spectrum exactly while leaving two ratios free. Equal rank-two event blocks in the finite tracial Gelfand–Naimark–Segal packet supply the balance $\rho/a = 1/\sqrt{2}$ exactly under the declared tracial premises; the Lean development checks the circulant identity and the equivalence [13].

Under the balance and mass-ordering premises, the measured electron and muon masses [60] determine the tau mass through one quadratic, and the comparison is governed by a registered conditional test whose target definition, precision floor of 0.045 MeV, and decision rule were fixed before comparison with any post-registration measurement [3]. The outward-rounded enclosure is [1776.968991, 1776.969063] MeV, a 72-eV window whose center sits 0.43σ from the measured 1776.93 ± 0.09 MeV [60]. The premise ancestry is declared: the balance premise was first abstracted from the measured triple, so this is a target-informed conditional postdiction whose evidential weight lies in the kill direction. A world-average or dedicated-measurement central value more than three standard uncertainties from 1776.969027 MeV refutes the balanced-circulant premise, and with it the equal-rank-two-block tracial reading of the face-circulant sector, the only exact family-sector mechanism this framework offers. The physical family attachment and phase selection are the open realization maps of this surface, so it is logically independent of the diagnostic below.

9.3 The charged-lepton interval diagnostic

A finite eight-path carrier combined with an empirical electromagnetic transport packet yields outward-rounded enclosures for the three charged leptons with logarithmic half-width 1.732%, one-sided multiplicative widths −1.72% and +1.75%, under the declared payload-coherent anchor-gap premise; the premise-free independent-gap enclosures have half-width 6.554%. The measured triple lies inside all three coherent enclosures [12]. Measured mass ratios and transport anchors enter this branch, so the result is a diagnostic of internal closure: it gives a sharp surface that a source-emitted transport bridge must hit, without claiming a source-only mass prediction. A physical charged determinant line, an absolute clock, and a source-emitted transport bridge are the maps an endpoint prediction requires.

9.4 The electroweak chart and strict pole-consumer checks

On the source-side branch, the source chain emits the electroweak chart coordinates 80.330 and 91.119 GeV for the W and Z carriers; no measured mass enters that emission [12]. Writing $S = gv_F/2$, $t = g'/g$, $w = S^2$, and $z = S^2(1+t^2)$, on the declared domain $S \neq 0$, $1 + t^2 \neq 0$, and $1 + d_Z \neq 0$, the strict one-loop consumer factorizes as

$$\frac{s_W}{s_Z} = \frac{1 + d_W}{(1+t^2)(1+d_Z)},$$

where $d_W = \Delta_{WW}(w)/w$ and $d_Z = \Delta_{ZZ}(z)/z$ are normalized self-energy factors. This is the exact quotient of the one-loop-truncated pole coordinates. Its strict one-loop expansion is $\frac{1}{1+t^2}(1 + d_W -$

$d_Z) + O(\text{loop}^2)$. The explicit common scale cancels under passive common-unit rescaling at fixed t, d_W, d_Z, while those three inputs remain unselected; an active change of v_F can move d_W and d_Z through thresholds. Directed complex-interval receipts exclude zeros of a scalar inverse-propagator entry on declared principal-sheet boxes. Separate receipts isolate, for each of W and Z, one simple scalar zero with derivative and scalar-residue balls in its declared lower-half pole box on a channel-specific algebraic chart [1]. The receipts identify neither chart with the physical resonance sheet and prove no unique continuation identity or sign bridge, full-matrix Laurent residue, physical current amplitude, or independent numerical replay. Their external Standard Model fixture is not composed with the source-emitted OPH chart coordinates. No chart-to-pole map, physical W/Z pole, or mass comparison follows from this stack.

9.5 Finite capacity and the de Sitter transfer sign

The screen's finite capacity supports an exact entropy accounting with one conditional gravitational reading [14].

Proposition 9.2 (Capacity transfer sign)**.** *For finite sector dimensions d_i with total $M = \sum_i d_i$ and sector probabilities p_i, the generalized entropy obeys the exact identity*

$$S_{\text{gen}}(p, d) = -\sum_i p_i \log p_i + \sum_i p_i \log d_i = \log M - D\left(p \,\middle\|\, \frac{d}{M}\right),$$

where D is relative entropy; its exact maximum is $S_{\text{gen}}^{\max} = \log M$ at $p_i = d_i/M$. If an observer receives a fraction $f \in (0, 1)$ of a fixed total capacity and the sectors deplete uniformly, every admissible transfer changes the maximum by

$$\Delta S_{\text{gen}}^{\max} = \log(1 - f) < 0,$$

and the positive-real interpolation in f is strictly decreasing and concave, so the zero-transfer point is a one-sided boundary maximum.

Proof. Expanding $D(p\|d/M) = \sum_i p_i \log p_i - \sum_i p_i \log d_i + \log M$ gives the identity, and nonnegativity of relative entropy with equality at $p = d/M$ gives the maximum. Uniform depletion replaces M by $(1-f)M$, so the maximum changes by $\log(1-f)$. Monotonicity and concavity of $f \mapsto \log(1-f)$ are elementary. □

Identifying capacity with horizon area, the transfer coordinate with observer mass, and the associated finite port operator with the gravitational shock supplies a conditional de Sitter time-advance mechanism with a definite sign: under those dictionaries an observer absorbing a capacity fraction f strictly lowers the maximal generalized horizon entropy by $\log(1 - f)$, and the one-sided boundary maximum carries the definite sign of the associated time shift. The finite identities make none of those physical identifications; the horizon and observer dictionaries are the open maps, and the finite calculation supplies no positive cyclic static-patch trace.

9.6 The unique rank-six invariant of the primitive carrier

In this subsection, "spin six" means spherical-harmonic angular rank $j = 6$. It does not denote a particle with spin six. The mathematical engine is the invariant count for the sixty-element proper icosahedral rotation group. The dimension of the invariant subspace in angular rank L is

$$m_L = \frac{1}{60}\Big[(2L+1) + 15\, U_{2L}(0) + 20\, U_{2L}(\tfrac{1}{2}) + 12\, U_{2L}(\tfrac{\varphi}{2}) + 12\, U_{2L}(\tfrac{\varphi-1}{2})\Big],$$

where U denotes the Chebyshev polynomials of the second kind. The exact table gives $m_1 = \cdots = m_5 = 0$ and $m_6 = 1$. Thus no nonconstant invariant occurs below angular rank six, and the rank-six invariant line is one-dimensional, in agreement with the classical icosahedral-harmonic classification [36].

Theorem 9.3 (Icosahedral angular-rank universality)**.** *Let the carrier kinetics be any operator $\lambda_a(k) = a^{-2}\sum_d w(d)\,[1 - \cos(a\,k\cdot d)]$ with a finite direction multiset and weights invariant under the proper icosahedral rotation group. Then the dispersion is exactly isotropic through angular rank five at every order in a; every directional term below angular rank ten is one multiple of the unique normalized invariant $\mathcal{I}_6$, whose stationary structure is the exact 62-direction census (12 and 20 extrema of opposite index, 30 saddles); and whenever the weighted sixth-moment $\mathcal{I}_6$ coefficient is nonzero, a condition certified for the equal-weight stencil and for each fundamental orbit, the first directional artifact appears at order a^4k^6 and one binary refinement step suppresses it by exactly 1/16 at that order. A vanishing sixth moment removes the displayed a^4k^6 residue. Any surviving directional term occurs at higher order. For a single-radius member whose weighted sixth-moment coefficient vanishes, the rank-six component vanishes at every order, so any surviving anisotropy has a higher even invariant rank. Rank six is the least symmetry-allowed nonzero anisotropic rank in the class.*

Proof. The order-$a^{2m-2}k^{2m}$ angular content is an invariant polynomial of degree $2m$, with harmonic components at even spins $L \le 2m$. The invariant table forces zero at $L = 2, 4, 8$ and a one-dimensional space at $L = 6$, so every anisotropic term below angular rank ten is proportional to $\mathcal{I}_6$, and the possible artifact ranks are exactly the even invariant levels $6, 10, 12, 16, \ldots$ The scaling in a is fixed by the expansion order, and the census is the exact critical-point theorem of the fingerprint certificate. The invariant table, the census, the constructive orbit verification, the nonvanishing sixth-moment coefficients, and an exact positive-weight tuned member whose spin-six content cancels are certified in exact arithmetic [1]. □

The universality theorem concerns a class of finite operators. A physical prediction requires a narrower branch whose operator and transfer are fixed. For the declared equal-weight twelve-port cosine member, set $x = |ak|$. On $0 < x \le 1$, the anisotropic part through eighth order is

$$A(x)\mathcal{I}_6(n), \qquad A(x) = \frac{2x^6(30 - x^2)}{118125} > 0.$$

If E_x denotes the complete x^{10}-and-higher angular tail after division by $A(x)$, exact spherical bounds give

$$\|\nabla E_x\| \le \frac{6875}{101152} < \frac{7}{100}, \qquad \|\operatorname{Hess}_{S^2} E_x\| \le \frac{383125}{562658} < \frac{7}{10}.$$

The 62 critical directions of $\mathcal{I}_6$ define 31 projective axes with minimum squared-sine separation $1/2 - \sqrt{5}/6$. Three exact projective charts, a 2,624-leaf interval cover, and local singular-value bounds prove that the full cosine kernel has exactly these 62 stationary directions for every $0 < x \le 1$. The twelve port directions remain maxima, the twenty face-center directions remain minima, and the thirty edge-center directions remain saddles [1]. Let $u_1, \ldots, u_{12}$ be the unit vectors through the twelve carrier ports. The *primitive twelve-port propagation branch* has the following premises. Its intrinsic spatial kinetic symbol belongs to the real, reciprocal, finite-range cosine class of Theorem 9.3, with scale-independent coefficients. The complete primitive orbit is its sole hop support, and no independent isotropic or directional kinetic term contributes through order k^6. Proper-carrier covariance acts on this orbit; the carrier scale a is finite and positive; the continuum quadratic term is normalized to k^2; one carrier rest frame is transported coherently over the experiment; and the tested scalar or polarization-independent sector realizes the operator. A photon realization requires equal action on both transverse polarizations. An empirical fit must isolate the carrier contribution from source, medium, gravitational, and instrumental effects. Transitivity of the proper carrier group forces equal weights once the single primitive orbit is selected. The carrier itself is realized as a quasiperiodic structure or as a graph: no periodic three-dimensional lattice carries this point group, so the branch presupposes no crystal.

The finite repair packet does not make this branch unavoidable. Its certified operator acts on thirty internal seam readings. A spatial translation stencil acts on a field at distinct sites, and a physical-sector readout is a third typed object. An exhaustive classification of the simulator's declared serialized finite artifacts finds local-domain kinetic operators and the twelve-port response on separate domains. It finds no registered accepted packet joining a complete twelve-port translation operator to a physical readout of that same operator. Exact equal-weight spatial stencils on the twelve vertex, twenty face, and thirty edge direction orbits have pairwise distinct rank-six rays. Proper-carrier transitivity fixes equal weights within any chosen orbit and does not select one orbit as the physical hop support.

Proposition 9.4 (Primitive twelve-port branch prediction)**.** *Under these premises, the physical symbol is*

$$\omega^2(k,\widehat{k}) = \frac{1}{2a^2}\sum_{i=1}^{12}\left[1-\cos\Big(ak\,u_i\cdot\widehat{k}\Big)\right]. \tag{20}$$

With

$$\mathcal{I}_6(n) = \frac{25}{132}\sum_{i=1}^{12} P_6(u_i\cdot n), \qquad \mathcal{I}_6(u_i)=1,$$

where P_6 is the degree-six Legendre polynomial. The small-ak expansion is

$$\omega^2 = k^2 - \frac{a^2}{20}k^4 + \frac{a^4}{840}k^6 + \frac{2a^4}{7875}k^6\mathcal{I}_6(\widehat{k}) + O(a^6k^8). \tag{21}$$

Writing the same expansion in a transported frame as

$$\omega^2 = k^2 + C_4k^4 + B_0k^6 + B_6k^6\mathcal{I}_6(R^{-1}\widehat{k}) + O(k^8), \qquad R\in \mathrm{SO}(3)/A_5,$$

the branch predicts

$$C_4 = -\frac{a^2}{20}, \qquad B_0 = \frac{a^4}{840}, \qquad B_6 = \frac{2a^4}{7875}, \tag{22}$$

and hence the scale-free relations

$$\frac{B_6}{C_4^2} = \frac{32}{315}, \qquad \frac{B_0}{C_4^2} = \frac{10}{21}, \qquad \frac{B_6}{B_0} = \frac{16}{75}. \tag{23}$$

All intrinsic anisotropic coefficients at $1\le j\le 5$ vanish. Once a negative C_4 is resolved on this branch, no scale or anisotropic amplitude is free; only one three-parameter orientation class R in $\mathrm{SO}(3)/A_5$ is profiled. Under binary refinement, $C_4(a/2) = C_4(a)/4$ and $B_0(a/2) = B_0(a)/16$, $B_6(a/2) = B_6(a)/16$ at fixed physical momentum.

Proof. The exact carrier moments are

$$\sum_i (u_i\cdot n)^2 = 4, \qquad \sum_i (u_i\cdot n)^4 = \frac{12}{5}, \qquad \sum_i (u_i\cdot n)^6 = \frac{12}{7} + \frac{64}{175}\mathcal{I}_6(n).$$

Substitution into the cosine series in (20) gives (21). Eliminating a gives (23). The lower-rank nulls and uniqueness of the rank-six line follow from Theorem 9.3. □

Proposition 9.5 (Source-seam edge ray and conditional propagation branch)**.** *The boundary current of the complete thirty-seam source packet maps through the antipodal-odd load readout onto*

$$D_6 = \left\{ z \in \mathbb{Z}^6 : \sum_i z_i \equiv 0 \pmod 2 \right\},$$

with Smith invariants $(1, 1, 1, 1, 1, 2)$*. Equip this module with the pullback of the response-selected Gram metric. Its completion is the same three-dimensional Euclidean carrier as the signed source-load module. This statement does not use the usual six-dimensional lattice metric on* D_6*. Every directed seam defines one translation of the internal record carrier, and the complete normalized direction multiset is the thirty-direction edge orbit. Cumulative records act simply transitively by internal isometries. If the feasible move laws and their objective are natural under the proper carrier action and A3 supplies a unique normalized minimizer, transitivity forces weight* $1/60$ *on every directed seam. The selected internal operator is the source-counting homogeneous convolution.*

Suppose these record translations are physical displacements under one homogeneous position action. Suppose also that the complete edge orbit is the sole direct support through the displayed order, proper-carrier covariance fixes one common weight, the same operator acts on one scalar or polarization-independent physical sector, its quadratic term is normalized to k^2*, and the action carries the stated gluing, scale, frame, readout, nuisance, and exclusivity data. Its spatial kinetic symbol is then*

$$\Lambda_a(k, \widehat{k}) = \frac{1}{5a^2} \sum_{j=1}^{30} \left[1 - \cos\!\left(ak\, w_j \cdot \widehat{k} \right) \right], \tag{24}$$

and its expansion has

$$C_4 = -\frac{a^2}{20}, \qquad B_0 = \frac{a^4}{840}, \qquad B_6 = -\frac{a^4}{12600}. \tag{25}$$

Consequently,

$$\frac{B_0}{C_4^2} = \frac{10}{21}, \qquad \frac{B_6}{C_4^2} = -\frac{2}{63}, \qquad \frac{B_6}{B_0} = -\frac{1}{15}. \tag{26}$$

The intrinsic anisotropic coefficients at angular ranks one through five vanish, and the rank-six vector lies on the rotated $\mathcal{I}_6$ *orbit.*

Proof. Exact integer incidence gives the displayed image and Smith invariants. The response-selected Gram pullback gives the dense three-dimensional completion. Every seam difference has squared norm four in the raw moment chart. The same full seam event has squared norm $2-2/\sqrt{5}$ in the unit-diagonal response-Gram completion. This common metric normalization does not supply a physical length. The sixty directed labels map two-to-one onto the thirty signed edge directions. The unit-normalized edge moments are

$$\sum_j (w_j \cdot n)^2 = 10, \qquad \sum_j (w_j \cdot n)^4 = 6, \qquad \sum_j (w_j \cdot n)^6 = \frac{30}{7} - \frac{2}{7}\mathcal{I}_6(n).$$

Substitution in the cosine series of (24) gives (25); elimination of a gives (26). The source incidence, quotient, orbit binding, and moment identities are machine-checked in exact arithmetic [1]. □

The proposition contains an exact finite source theorem and a conditional physical branch. The conditional naturality and minimizer theorem supplies a homogeneous internal action. Its equal source-counting average P preserves constants and positivity. The generator $L = I - P$ satisfies the positive maximum principle and the exact local identity

$$2fLf - L(f^2) = \frac{1}{60}\sum_e \big(f(x+v_e) - f(x)\big)^2 \ge 0. \tag{27}$$

Plane waves diagonalize L. If a_{edge} is the exact internal step norm, the exact response-coordinate character is

$$\Lambda_{\mathrm{int}}(\mathbf{k}) := \frac{6}{a_{\mathrm{edge}}^2}\lambda_L(\mathbf{k}).$$

It equals the edge-current character evaluated at the internal response step. An auxiliary algebraic phase lift $(q,p) \mapsto (p, -Lq)$ gives $q'' + Lq = 0$ and $\omega_{\mathrm{aux}}^2 = \lambda_L(k)$ on each plane wave. The primes and ω_{aux} name algebraic phase coordinates and a spectral root. This theorem does not construct trajectories, a physical clock, a conserved energy, or a physical field equation. A physical dilation that maps the internal step to a length a would give

$$\Lambda_a(k,\widehat{k}) = \frac{6}{a^2}\lambda_L\left(\frac{a}{a_{\mathrm{edge}}}k\widehat{k}\right),$$

where $\mathbf{k} = k\widehat{k}$, $k = |\mathbf{k}|$, and $|\widehat{k}| = 1$. The declared finite spatial symbol can be controlled without the physical frequency premise. Write $q = ak$ and

$$\widehat{\Lambda}(q,n) = \frac{1}{5}\sum_{e=1}^{30}[1 - \cos(q\, w_e \cdot n)].$$

The separately kernel-checked eighth moment is $M_8 = 10/3 - (8/15)\mathcal{I}_6(n)$. With

$$P_6 = q^2 - \frac{q^4}{20} + \left(\frac{1}{840} - \frac{\mathcal{I}_6}{12600}\right)q^6,$$
$$P_8 = P_6 + \left(-\frac{1}{60480} + \frac{\mathcal{I}_6}{378000}\right)q^8,$$

the exact range $-5/9 \le \mathcal{I}_6 \le 1$ and alternating cosine bounds give, uniformly for $0 \le q \le 1$,

$$|\widehat{\Lambda} - P_6| \le \frac{7}{388800}q^8, \qquad 0 \le \widehat{\Lambda} - P_8 \le \frac{7}{34992000}q^{10}, \qquad \frac{19}{20}q^2 \le \widehat{\Lambda} \le q^2.$$

This target-free certificate concerns the mathematical spatial operator. It does not turn a into an SI length or select a physical field or clock. There is also a basis-free transverse oscillator completion of this spatial operator. At every nonzero momentum, the momentum-orthogonal fiber has real dimension two. The transverse projector is idempotent, the scalar symbol acts equally on the complete fiber, and the declared first-order generator gives $A_T'' + \Lambda_a A_T = 0$, $\omega_a^2 = \Lambda_a$, and $\omega_a(0) = 0$. The corresponding quadratic energy has zero algebraic first variation on that generator. The Maxwell-shaped first-order structure is composed rather than adjacent: an explicit boundary map carries the supplied mode dynamics onto the opposite-sign curl pairing at the identified frequency, exactly, on every transverse state at every nonzero chart momentum, with both modal divergence constraints preserved and the same-sign mutation failing the wave law; the intertwining is a factorization, not an equivalence. The finite gauge object the boundary lists named as missing is also committed and classified: $U(1)$-valued seam connections modulo port gauge transformations are classified exactly by their nineteen chord holonomies—the first cohomology of the thirty-seam graph with circle coefficients is $U(1)^{19}$, the same count as the source-free Gauss cycle rank, through the eleven-seam spanning tree of the committed boundary section. These statements depend on choosing the spatial symbol as the oscillator stiffness. They do not construct a position-space flow, dynamics for the committed connection, a physical clock, a photon Hilbert space, or a massless physical particle.

An exhaustive target-free calibration enumerates all 2^{12} sign-error vectors in one fixed twelve-row design. At noise scale 1/200, the leading coefficient is detected in all 4096 replicas, while the nominal 95 percent interval covers its injected value in 3904 replicas. The linked isotropic and rank-six higher-order pair is detected in 209 of 4096 replicas and is unresolved in this stress law. The exact spatial-symbol remainder cannot erase the leading detection margin in this design. This result supplies no continuous-direction coverage, detector response, nuisance model, five-standard-deviation tail calibration, or statement about experimental sensitivity.

This dilation is not a physical clock calibration. If Λ_a is separately identified with physical ω^2_{phys}, the positive leading formal branch gives

$$\omega = k - \frac{a^2}{40}k^3 + a^4\left(\frac{19}{67200} - \frac{\mathcal{I}_6}{25200}\right)k^5 + O(a^6k^7), \tag{28}$$

$$\frac{\partial\omega}{\partial k} = 1 - \frac{3a^2}{40}k^2 + a^4\left(\frac{19}{13440} - \frac{\mathcal{I}_6}{5040}\right)k^4 + O(a^6k^6), \tag{29}$$

$$\frac{1}{k}\nabla_{S^2}\omega = -\frac{a^4}{25200}k^4\nabla_{S^2}\mathcal{I}_6 + O(a^6k^6). \tag{30}$$

These are exact formal-series coefficients derived without comparison data. The spatial-symbol bound does not provide an analytic remainder for the physical-frequency branch, physical position, photon or other field sector, clock, cofinal gluing, physical scale, preferred frame and boost law, wave-packet dynamics, laboratory readout, or nuisance model needed for a time-of-flight or other physical comparison. The edge branch and the primitive-port branch have opposite rank-six signs. Their coefficient rays, custody packages, exposure exclusions, and comparison budgets remain separate.

One exact theorem binds the two frozen rays and every member of the declared positive-weight scalar cosine class into a single test surface.

Theorem 9.6 (Carrier-class dispersion band)**.** *Consider the declared positive-weight scalar cosine class of spatial symbols*

$$\Lambda(k,\widehat{k}) = \sum_s w_s \sum_{u\in O_s}\left[1-\cos(ak\,r_s\,u\cdot\widehat{k})\right],$$

where $a>0$, each O_s is one orbit of the sixty proper carrier rotations on unit directions, the finitely many shells carry radii $r_s>0$ and weights $w_s>0$, the displayed cosine sum is the full spatial symbol and hence complete through order k^8, and the continuum normalization fixes the k^2 coefficient to one. Write the normalized expansion as

$$\Lambda_{\mathrm{norm}} = k^2 + C_4k^4 + (B_0 + B_6\mathcal{I}_6(\widehat{k}))k^6 + (D_0 + D_6\mathcal{I}_6(\widehat{k}))k^8 + O(k^{10})$$

and set $\mu_m = \sum_s w_s|O_s|r_s^m$. Then five exact statements hold.

1. *Sign law: $C_4 = -(a^2/20)\,\mu_4/\mu_2 < 0$ for every member.*

2. *Isotropic floor: $B_0/C_4^2 = (10/21)\,\mu_2\mu_6/\mu_4^2 \geq 10/21$, with equality exactly on the single-radius members.*

3. *Rank purity: the intrinsic anisotropic ranks one through five vanish and the rank-six residue is one multiple of the rotated $\mathcal{I}_6$.*

4. *Rank-six band: every member has* $B_6/B_0 = (16/75)\,\langle \mathcal{I}_6(\widehat{u}_s)\rangle$*, the mean taken with the positive weights* $w_s|O_s|r_s^6$*, and this ratio lies in* $[-16/135,\ 16/75]$ *for the declared class. The pure face and vertex orbits attain the endpoints, the pure edge orbit sits at* $-1/15$*, and the vertex-face mixture at per-direction weight ratio* 25:27 *attains zero.*

5. *Eighth-order confinement and common-radius lock: every member has*

$$D_0 = -\frac{a^6}{60480}\frac{\mu_8}{\mu_2}, \qquad \frac{D_6}{D_0} = \frac{64}{125}\,\langle \mathcal{I}_6(\widehat{u}_s)\rangle_{w_s|O_s|r_s^8}\,.$$

No independent angular shape occurs at order k^8*. Every member whose active shells share one radius obeys the division-free identity* $5D_6B_0 = 12B_6D_0$*. This includes the* 25:27 *zero-anisotropy mixture, where both sides vanish.*

Proof. The group-summed sixth-power kernel factors on the invariant line: for every seed direction u,

$$\sum_g \big((gu)\cdot\widehat{k}\big)^6 = \frac{60}{7} + \frac{64}{35}\,\mathcal{I}_6(\widehat{u})\,\mathcal{I}_6(\widehat{k})$$

on unit vectors, the sum over the sixty proper rotations. The kernel is invariant in the seed, the recomputed invariant multiplicities $(1,0,0,0,0,0,1)$ through rank six pin the degree-six invariant space to the span of the radial power and the $\mathcal{I}_6$ form, and exact residue-zero verification at seeds with distinct $\mathcal{I}_6$ values determines the identity on that two-dimensional space. The moment ratios then give the displayed C_4 and B_0, and the floor is the Lagrange identity. Writing $W_s = w_s|O_s|$, it is $\mu_2\mu_6 - \mu_4^2 = \sum_{i<j} W_iW_jr_i^2r_j^2(r_i^2 - r_j^2)^2 \geq 0$, which vanishes exactly at one radius. The band follows from the kernel factorization: the rank-six numerator is the positive-weighted sum of seed $\mathcal{I}_6$ values against the matching isotropic denominator, so every member averages inside the range of $\mathcal{I}_6$, which the 62-direction stationary census places at $[-5/9, 1]$ with the orbit values 1, $-5/16$, and $-5/9$. At degree eight, the invariant multiplicities $(1,0,0,1,0)$ at ranks $0,2,4,6,8$ and the exact kernel identity

$$\sum_g \big((gu)\cdot\widehat{k}\big)^8 = \frac{20}{3} + \frac{256}{75}\,\mathcal{I}_6(\widehat{u})\,\mathcal{I}_6(\widehat{k})$$

leave the radial term and the rotated $\mathcal{I}_6$ as the complete through-eighth-order basis. Normalizing the cosine coefficient by its quadratic term gives the displayed D_0 and weighted-mean formula. At one common radius the sixth- and eighth-order means agree, and $(64/125)/(16/75) = 12/5$, which proves the polynomial lock without dividing by either anisotropy. A second exact series route and an independent high-precision fit of the raw cosine symbols reproduce the same table. One-sign nonnegativity is load-bearing: a signed-weight control member violates the floor, while active shells carry strictly positive weights. The exact constants, finite seed controls, and series checks are machine-checked. The universal class statements follow from the displayed invariant-space and Lagrange arguments [1]. □

The theorem converts the two frozen branches into two points of one exact map. The declared class shares a support-independent negative quartic sign and the floor $B_0/C_4^2 \geq 10/21$. The excess above the floor is a normalized weighted radial-moment gap and vanishes exactly on common-radius support. The signed rank-six ratio constrains the weighted angular moment and distinguishes the three pure fundamental orbit rays. FZ-11 sits at the vertex endpoint $16/75$ of the band and FZ-12 at the edge point $-1/15$. The class-level kill surface is correspondingly wider than either branch band: under the same physical-sector premises, a resolved intrinsic dispersion with B_0/C_4^2 below $10/21$, or B_6/B_0 outside $[-16/135, 16/75]$, or a residue at ranks one through five, or a rank-six residue off the rotated $\mathcal{I}_6$ template, excludes every member of the declared positive-weight scalar cosine class at once. Exact saturation of the floor at 10/21 certifies common-radius support; a finite-precision result near the floor bounds the corresponding Lagrange gap normalized by μ_4^2. A generic rank-six anisotropic model carries independent k^6 and k^8 amplitudes. The common-radius carrier stratum fixes their relative amplitude, while multi-radius members retain correlated radial-moment dependence. The isotropic tower alternates in sign at every order. The certificate reads no comparison data and changes no frozen bytes [1].

The edge ray and its complete decision rule are frozen under public commit custody before any eligible edge-branch comparison payload [3]. The WMAP template class, every primitive-port comparison input, and every comparison datum inspected before the edge freeze are excluded. Physical comparison is ineligible while a promotion premise is open. A null has no verdict unless a pre-exposure source theorem for the same action and sector supplies $a_{\min} > 0$, and a preregistered power and remainder contract makes the entire admitted $a \geq a_{\min}$ manifold excludable by the joint-likelihood rule.

Conditional photon and pair-production kinematics. If the seam symbol is identified with physical photon frequency squared in the Euclidean carrier metric, its complete cosine form satisfies

$$0 \leq \Omega_\gamma(k, \widehat{k})^2 = \Lambda_a(k, \widehat{k}) \leq k^2$$

for every nonzero a. With ordinary additive energy-momentum conservation in the declared frame and positive-mass electrons and positrons with Lorentz-invariant positive-energy dispersion, this bound excludes photon decay into an electron-positron pair. At fixed incoming momenta, the seam-current incoming-energy budget is contained in the Lorentz-invariant photon budget. These kinematic statements supply no interaction vertex, cross section, opacity, event rate, source population, shower development, or detector response.

Use the leading dispersion convention $E_i^2 = p_i^2 + m_i^2 + \delta_{i,2}E_i^4$, with $i \in \{\gamma, +, -\}$, $m_\gamma = 0$, $m_+ = m_- = m_e$, E the hard-photon energy, and ϵ the soft-photon energy. With the soft background photon Lorentz invariant at leading order, independent leading dimension-six coefficients for the hard photon, positron, and electron, together with a fixed positron energy share $0 < x < 1$, give the head-on, collinear equation

$$\left[\delta_{\gamma,2} - x^3\delta_{+,2} - (1-x)^3\delta_{-,2}\right] E^4 + 4\epsilon E - \frac{m_e^2}{x(1-x)} = 0.$$

At equal sharing, the observable coefficient is $\delta_{\gamma,2} - (\delta_{+,2} + \delta_{-,2})/8$. The corresponding linear map has rank one and a two-dimensional coefficient fiber. A photon-only threshold bound therefore requires the stated Lorentz-invariant-lepton premise or a source derivation of the charged-lepton actions. On the Lorentz-invariant-lepton branch, exact algebra proves that equal sharing uniquely minimizes the mass penalty and globally maximizes this leading head-on, collinear residual. General independent-lepton share optimization and the full anisotropic minimization over outgoing momentum remain separate problems [1].

Exposed source-seam diagnostics. The external subluminal quadratic-dispersion convention identifies $a = \sqrt{20}/E_{\mathrm{QG},2}$ on the conditional edge branch, where $E_{\mathrm{QG},2}$ denotes the external quadratic dispersion scale. A published time-of-flight bound $E_{\mathrm{QG},2} > 10^{13}\,\mathrm{GeV}$ gives $a < 8.825 \times 10^{-29}\,\mathrm{m}$ at 95 percent confidence; the Large High Altitude Air Shower Observatory (LHAASO) bound $E_{\mathrm{QG},2} > 6.9 \times 10^{11}\,\mathrm{GeV}$ is a cross-check with a different source-lag treatment [57, 58]. A more sensitive but source-model-dependent Pierre Auger diagnostic uses $\delta_{\gamma,2} = -a^2/20$ [56]. Its alternative source scenario with a subdominant proton component extending to $10^{20}\,\mathrm{eV}$ reports $\delta_{\gamma,2} > -10^{-58}\,\mathrm{eV}^{-2}$, which gives

$$a < \sqrt{20}\,10^{-29}\,\mathrm{eV}^{-1} = 8.825 \times 10^{-36}\,\mathrm{m} = 0.546\,\ell_P.$$

This translation assumes Lorentz-invariant electrons and positrons, additive conservation in the preferred cosmic-background frame, photon-only Lorentz violation, a threshold-shifted Breit–Wheeler treatment, and the named source scenario. The reference scenarios give no electromagnetic constraint, the direct bound has no stated confidence level, and the shower response was not recomputed under the modified dynamics. The exact $0 \leq ak \leq 1$ spatial remainder applies only after the physical identifications are made. These exposed limits are conditional diagnostics of an open scale. They carry no OPH verdict or evidence weight.

Primitive-port prediction and decision rule. Proposition 9.4 is a frozen prospective conditional physical-branch prediction [3]. The minimal locally Lorentz-invariant Standard Model plus General Relativity in local vacuum supplies the baseline $C_4 = B_0 = B_6 = 0$ for intrinsic propagation. Nonminimal effective operators, media, curvature, or another icosahedral system can imitate some or all of the pattern. A match therefore distinguishes the branch from the minimal baseline without identifying OPH uniquely. Searches for Lorentz violation commonly organize anisotropic coefficients by angular rank j in the Standard-Model Extension [54, 55], which provides a natural coefficient language for a dataset-specific contract: at leading order the branch populates only the thirteen-component nonbirefringent rank-six multiplet at operator dimension eight, collapsed to one amplitude and one frame orientation, with every anisotropic coefficient at $1 \leq j \leq 5$ exactly zero in the carrier rest frame and frame boosts admixing neighboring ranks at first order in the frame velocity.

The exposure class is fixed. The WMAP internal-linear-combination map, its cosmic-microwave-background likelihood class, and every data product examined in the dated template search are ineligible for this prediction. That search included the icosahedral rank-six and higher templates and returned a family-wide p-value of 0.64. The linked C_4, B_0, and B_6 relations have no eligible physical comparison. The target statement, exact coefficient snapshot, registration manifest, public commit custody, and detached OpenTimestamps calendar attestations fix the prospective content and its decision rule [3].

No empirical verdict is licensed without a dataset-specific registration that fixes one post-freeze release, a joint likelihood or full covariance for the same-sector C_4, B_0, and rank-six coefficient vector, the carrier and observer frames, the $\mathrm{SO}(3)/A_5$ orientation profile, the boost law, the environmental and instrumental nuisance model, trials accounting, sensitivity floor, and calibrated joint threshold. The decision rule scores the complete branch manifold

$$C_4 < 0, \qquad B_0 = \frac{10}{21} C_4^2, \qquad B_6 = \frac{32}{315} C_4^2,$$

with every intrinsic coefficient at $1 \leq j \leq 5$ equal to zero and the $j = 6$ vector on the rotated $\mathcal{I}_6$ orbit. The branch fails at five standard deviations or more if an isolated intrinsic C_4 is positive, an isolated lower-rank coefficient is nonzero, the linked sixth-order terms are excluded at adequate sensitivity, or the calibrated profile likelihood excludes the complete manifold. Support requires exclusion of the zero-coefficient baseline at five standard deviations or more, agreement with the linked branch manifold within two standard deviations, rejection of named systematic alternatives, and an independent replication. A null result is inconclusive because the branch supplies no positive lower bound on a. Incomplete covariance, insufficient sixth-order sensitivity after a negative C_4, or failure to isolate the carrier contribution is also inconclusive. A failed verdict rejects the primitive twelve-port physical propagation branch. The certified source does not supply a spatial translation operator, complete propagation grammar, same-operator physical readout, or coherent frame and boost law. A failure applies to OPH as a whole only after a stronger source law makes this branch forced and exclusive.

Boundary 9.7. No statement in this section is a source-only determination of a physical constant or mass. The Koide identity and capacity identities are finite theorems; the tau window is a registered conditional test, postdictive in premise ancestry; the lepton enclosures and the screen-grain separation are diagnostics that consume measured inputs; the electroweak chart and external scalar-pole fixture are uncomposed diagnostics and define no W/Z pole comparison; the de Sitter reading is conditional on unconstructed dictionaries. The angular-rank theorem is a finite theorem. The primitive-port and source-seam coefficient relations form separate prospective predictions on their named physical branches, with empirical scoring sealed until the required branch-specific contracts exist.

10 Physical interpretation maps and their consequences

The preceding theorems concern finite records, explicit response maps, and reconstruction implications. They become claims about nature only when a physical carrier realizes the operations and identifications in Definition 1.2. Table 3 records the resulting implications and the premise that can fail.

Table 3: Physical questions, reconstruction consequences, and required premises.

Question	**Consequence of the stated realization**	**Physical premise or open map**
When is a measurement public?	A completed record is a schedule-independent normal form; on the declared algebra-state surface its event projectors obey Theorem 4.1.	A laboratory system must instantiate durable records, readback, protected boundaries, and the algebra-state map.
When is event geometry Lorentzian?	A common record-germ tower satisfying (E1)–(E7) gives the Lorentzian four-manifold of Theorem 6.2; the finite instrument separately measures inertia $(1,3)$ on its declared path.	A physical refinement family must supply the common-refinement pseudometric, population, covering charts, compatible tetrads, cone, and causal premises.
What equation governs the gravitational response?	A realized null balance, Ward conservation, the Bianchi identity, and an independent scale identification give the Einstein field equation of Theorem 7.2.	The common physical tower, continuum control, clock, stress, area, vacuum, and scale maps are the open realization maps.
How do the gravitational and gauge branches share a source?	One carrier can feed the event/modular branch and the twelve-port response branch, yielding both conclusions from a common source.	Joint physical realization and equality of G_{packet} with G_{Tan} are separate identification premises.

Table 3 continued

Question	Consequence of the stated realization	Physical premise or open map
Why this gauge type and matter image?	The complete A1 response and endogenous A2 transport force the compact local Lie type. The separately supplied matter representation gives the conditional $\mathbb{Z}_6$ quotient and anomaly-free rank-fifteen finite module.	Laboratory currents, quantum fields, physical matter poles, family multiplicity, and masses require further maps.
What propagation signature distinguishes the primitive-port branch from the minimal locally Lorentz-invariant baseline?	Proposition 9.4 fixes B_0 and B_6 from C_4 and fixes the complete rank-six shape up to one carrier orientation.	The internal seam-repair operator is not a spatial hop operator. The scalar or polarization-independent sector bridge, coherent frame transport, carrier-term isolation, and exclusivity are branch premises.

10.1 Gravity through the modular-entropy map

The route runs from patches to gravity in one chain: consensus supplies public record classes, a record-germ realization satisfying (E1)–(E7) supplies the event manifold, modular flow and half-sided inclusions supply the local time generator and null translations on the same tower, and the entropy split with its stationarity conditions would then have to establish the null-balance premise that Theorem 7.2 completes. This is a route with a visible break point. The normal-form, entropy, and tomography statements have exact finite layers. The passage to gravity starts only when one physical tower realizes the event, modular, continuum, universal-coupling, vacuum, and scale premises together.

Intuition. Under the stated realization map, the gravitational response represents the large-scale compatibility condition associated with repaired records. Local observers extend and compare records; the metric describes the stable causal relation among those records. Modular flow describes how a patch reads change, and entropy stationarity fixes the response needed for the local accounts to stay compatible.

10.2 A common source for gravitational and gauge branches

In this framework, the proposed fundamental theory that combines the gravitational and gauge sectors is one observer-consensus tower with two readout branches. The overlap, record, and repair structure feeds the event and modular branch. The twelve-port incidence, complete reversible response, and endogenous overlap transport feed the local gauge-type branch. Trace-balanced blocks supply a separate conditional matter branch. A single self-reading carrier could supply both sets of antecedents.

This common source neither identifies gravity with a gauge boson nor fixes all couplings. It also leaves two gauge constructions distinct: G_{Tan} is reconstructed from a sector category, while G_{packet} is the maximal faithful image of the explicit finite response and matter module. A completed unification requires joint physical realization and a proof that the corresponding current diagrams commute.

Intuition. One object performs two tasks. Its overlap structure organizes events and the gravitational response; its boundary response organizes charges. The proposed architecture takes the carrier and its self-reading dynamics as their common source, with spacetime and gauge structure appearing as distinct public readouts.

10.3 Completed records as measurement events

The finite construction locates a measurement at the transition from a private, repairable record to a protected public normal form. Event projectors, weights, and state updates are then defined on the completed algebra-state surface. A physical application must identify actual durable records and verify the transaction and readback premises. The theorem does not construct the state space of an interacting continuum field theory.

Intuition. Within the model, a record is called public when it has survived every authorized comparison and no repair can change it. Probability describes the completed record surface; state update moves from one public record class to the next. The construction locates observation in the bookkeeping that patches perform on their shared boundaries.

10.4 Thermodynamics from conditional repair

The four laws form a finite conditional theorem package, by short elementary arguments once Axiom 3 is read on transition distributions and the five typed source and physical receipts below are supplied. Axiom 3 applies separately to states and to transition distributions, and both information projections are solved exactly. The state projection with faithful reference and conserved constraints is the Gibbs exponential family, by the information-projection Pythagorean identity. On one supplied nondegenerate finite spectrum, equality of Gibbs distributions identifies the inverse temperature; this is the finite zeroth-law transitivity receipt, not a contact dynamics or equilibration theorem. The transition projection onto the fibre of the complete repaired visible datum is weighted conditional resampling from the same reference, the weighted observation-fiber projector of the consensus construction. That kernel is stochastic, idempotent, reversible, and stationary, fixes every fibre-measurable charge, and contracts relative entropy to the reference: the second law is a data-processing theorem for repair, with the modular form $\Delta S \geq \Delta\langle K\rangle$, $K = -\log \tau$, on τ-preserving channels, and the Landauer erasure bound as a corollary; the inequality is the mean of a fluctuating entropy production that obeys exact integral, pointwise, and level-set fluctuation identities. The mean is an identity rather than a bound: one repair step's mean entropy production equals the relative entropy from the input state to its repaired image exactly, so it is strictly positive whenever the step changes a strictly positive state and vanishes precisely on the fibre-conditional-reference fixed points. This is a strict single-step orientation off the repaired manifold. The identical kernel is idempotent and therefore produces zero further dissipation after the first projection; a sustained macroscopic arrow is not claimed. The whole package is one machine-checked theorem: a single typed antecedent bundle—the declared repair law, the clock-and-energy calibration, and the refinement-uniform gap family—yields one conclusion record whose every clause, from the zeroth law to Landauer and the refinement-uniform third law at the same calibrated energy, consumes that bundle. Detailed balance plus a linear Poisson solver on centered currents gives a symmetric positive-semidefinite finite Green–Kubo matrix with an exact cutoff remainder [24, 25]. Separately, for each current pair, iterating the idempotent full-fibre time-step gives either zero positive-lag correlation or nonstabilizing partial sums, so a nonzero decaying memory tail with a stabilizing Green–Kubo sum requires a separately sourced nonidempotent evolution. For a finite joint update, the exact bookkeeping split is $\Delta U = \delta Q + \delta W + \operatorname{Re}\operatorname{tr}(\delta\rho\,\delta H)$; the two-term form is first-order or exact for an ordered fixed-H or fixed-ρ stroke. The composed repair heat stroke holds H fixed. At every finite regulator the excited Gibbs mass is bounded by $\frac{d-g_0}{g_0}e^{-\beta\Delta}$. The entropy limit $k_B \log g_0$

is a standard finite corollary of that bound rather than a separate field of the composed Lean record. Finite-step unattainability follows because faithful repair and pinching steps cannot reach the rank-deficient zero-temperature state. The exact identity $S(p) - S(\tau) = \langle K\rangle_p - \langle K\rangle_\tau - D(p\|\tau)$ makes the entanglement first law $\delta S = \delta\langle K\rangle$ a first-order corollary, and through the central split $K = 2\pi B + Z$ one repair step with a fibre-measurable central charge obeys the cap Clausius inequality $2\pi\Delta\langle B\rangle \leq \Delta S$; the Einstein branch's finite first-law premise package is discharged on the simplex tangent space of this model, with the physical split retained by the energy-clock receipt. The strict-descent normalizer that settles public facts is a different map and carries no entropy inequality; the explicit two-point counterexample is certified. The Lean modules carry the finite statements, and exact certificates replay the kernel algebra. The pinned source artifact has an exact negative verdict: its state-side resampling action is idempotent, while its recurrent transition action has a nonconstant eigenmode with $\lambda = 665437/726948 \in (0, 1)$, so every dynamic intertwiner kills that mode. Its stationary mass 7155/61511 is also not a deterministic pushforward of the 16384-sample state reference. Thus no nondegenerate action intertwiner or deterministic empirical pushforward binds this committed pair. The B20 exact preflight then exhausts one declared random-scan grammar: uniform-scheduler mixtures in both certified arenas are non-idempotent, while under both declared schedulers every computed fixed space is one-dimensional. Constant-field cases inherit only the fixed-space result. This excludes a nonconstant protected observable inside that grammar, not adaptive, zero-weight, dilated, enriched-export, or new-source routes. The conditional theorem requires a separately source-justified stochastic coupling or a different source object, together with a global objective, one nondegenerate shared reference and collar kernel, energy-clock calibration, and uniform low-temperature spectral-tail control on a coherent cofinal family. The current finite gap-plus-cardinality theorem is one sufficient witness, not a necessary continuum route [9, 1]. Under those named receipts, thermodynamics is recovered rather than imported.

10.5 Forced dynamics, the derived action, and the two faces of mechanics

Dynamics on the private block is the unique continuous completion of the record-preserving symmetries. Every pointwise-continuous one-parameter star-automorphism group of the finite private algebra is blockwise unitary conjugation generated by one time-independent self-adjoint Hamiltonian per Wedderburn block, unique up to a real scalar [9, 1]. Time evolution on this surface has no freedom beyond a Hamiltonian; the Schrödinger form is a theorem whose single premise, pointwise continuity in the flow parameter, is displayed.

The action of the realized history law is likewise derived. For any strictly positive row-stochastic kernel and strictly positive normalized initial law, the Markov path law is the exponential tilt of the step-uniform reference by the log-transition action at multiplier one, and an action-multiplier pair reproduces the law exactly when the multiplier-weighted action equals the log-transition action plus a constant, so the action is unique up to an additive constant and a multiplier rescaling. The bare-action convention chooses multiplier one; it is unique for a nonconstant path action, while a constant action leaves the multiplier invisible after normalization. The committed source chain instantiates both statements through kernel-decided integer receipts, and its committed repair-count action reproduces the chain law at no multiplier: the uniqueness is not vacuous. The declared object is the reference measure; given it, the dynamics selects its own action.

A finite Legendre bridge joins the two attained faces of mechanics. The discrete Euler–Lagrange condition at a junction holds exactly when one step of the discrete Hamilton flow carries the incoming junction state to the outgoing one, for the quadratic class and for strictly convex Lagrangians with a solver section; single-site minimizers of the log-transition local action coincide with most probable paths of the realized chain through an exact corner identity, so least action and maximum probability are two readouts of one functional; and the constant Noether current of the chain face equals the quadratic Legendre momentum, conserved together with the energy along the free Hamilton orbit that reproduces the committed witness path. The stronger reverse-direction audit is an exact non-identifiability theorem. The bilinear real extension of the source corner table is velocity-affine and has no global Legendre solver, while every $L_a = L_0 + \frac{a}{2}y(y-1)$, $a > 0$, agrees on every realized history yet is strictly convex with an explicit Hamiltonian; the checked $a = 1, 2$ members are distinct. Thus no theorem can produce a unique real Hamiltonian continuation from this finite history law without an additional curvature/source receipt.

The two faces form one machine-checked theorem: under the registered path reference, real enrichment, and supplied-dynamics rows, a fixed-endpoint single-site-extremal embedded history realizes the derived action of the same kernel whose exponential tilt is the path law, is an interior most-probable update of that law, and satisfies the discrete Hamilton equations, with the Noether momentum constant under the supplied symmetry data. Inside the declared one-parameter enrichment family, requiring the constant-one transition-weight maximizer to be stationary under every fixed-endpoint single-site variation forces the member uniquely, $a_\star = 2\log(W_{11}^2/(W_{10}W_{01}))$, the curvature being exactly twice the log of the interior mode-dominance ratio, and positive precisely because that dominance is strict. At $a_\star$, the same history is a fixed-endpoint single-site minimizer at every interior junction, so the composed conditional is inhabited on the committed kernel. The two-parameter corner-invisible counterfamily shows that this rule does not select a unique real enrichment beyond the one-parameter ansatz, and no global simultaneous multi-site minimum is claimed. The principle references off-alphabet variations and is declared; the non-identifiability theorem stands.

The strongest new finite discriminator uses only the pinned oriented incidence. A declared equal-weight cyclic rule on its twenty faces gives the A_5-equivariant bracket $B_{\text{face}} = 60R_{13}$. It is not a Lie bracket: exactly 240 of 2640 independent Jacobi coordinates are nonzero, split equally between $+1$ and -1. Exact primal–dual certificates compare it with the classified compact locus under three coordinate edit norms:

	G	F	P
ℓ_1	$30(\sqrt{5}-1)$	60 (infimum)	60
ℓ_2^2	$(615-123\sqrt{5})/22$	$(615+123\sqrt{5})/22$	45
ℓ_∞	$(5-\sqrt{5})/10$	$\sqrt{5}/5$	1/2.

Thus G wins all three comparisons. The equal-weight rule, coordinate norms, and minimum-distance repair principle are displayed premises; the theorem is a robust compact-family discriminator, not a source-selected Jacobi repair or a comparison with the entire Jacobi variety.

The reconstructed kinetic sector adds an independent exact separation. For carrier-projector quadratic forms, ad-invariance reduces the F/G three-weight space to dimension two:

$$F: \; w_{3+} = \sqrt{5}\, w_5, \qquad G: \; w_{3-} = \sqrt{5}\, w_5,$$

whereas the P control remains two-to-two. Imposing both mirror relations would leave one ray, but that simultaneous premise is not derived. The general invariant form has one coefficient per simple factor and no cross term. A symbolic two-factor theorem then binds the constructed Gibbs kernels for both P factors and the complete color-bearing F family. Equality of two such kernels leaves only common multiplier scaling, so the relative factor coefficient is identifiable but not selected. No independent OPH source kernel or coefficient value is obtained.

Boundary 10.1. These are representation-level statements over committed finite packets. The face rule, metric, compact-locus projection, reference measure, step lattice, quadratic cost, and Gibbs kernel are declared or constructed objects; no independent source selection, physical unit, clock, complex amplitude, continuum limit, or laboratory field is claimed. Relative coefficients between simple factors, the abelian kinetic ray, and the matter, scalar, and hypercharge sectors of any composed effective action remain gated on the frozen source-selection protocol, with mismatch against the frozen Standard Model target a valid exit.

10.6 Event geometry from record germs

The spherical support identifies the Lorentz group and the hyperbolic space of timelike directions without supplying physical events. Events enter only through the direct limit of separating record germs. Theorem 6.2 states a sufficient gluing criterion: four-component charts with open images, compatible affine transitions, tetrads, a Lorentzian quadratic form, and causal reachability must all occur on one populated refinement limit. The criterion packages these inputs into one Lorentzian manifold; it does not select four dimensions or Lorentz signature from weaker data. The finite instrument measures rank and inertia on a prescribed chart, while its negative cone margins and absent cofinal limit prevent continuum promotion.
Intuition. Record germs say when two finite histories describe the same event and how nearby events can be distinguished. A spacetime appears only when those local descriptions cover the completed event set, agree on overlaps, and carry one compatible causal cone. The manifold records the stable way in which public events fit together.

10.7 Gauge type and the finite matter image

The twelve-port coefficients split under the alternating-group action into bands of dimensions 1, 3, 3, and 5. A1 supplies their complete compact response algebra, while A2 implements every proper carrier action internally on that same response. Theorem 8.2 forces one central direction together with simple ideals of dimensions three and eight. The response map K is an exact conditional matrix witness for this algebra. The matter space V is a separate supplied representation; its exterior algebra gives the familiar fifteen-state branching, and the exhaustive scan proves the two-mask selection inside its declared ten-summand menu. Physical gauge fields, matter poles, families, and masses require current and continuum realization maps.
Intuition. The carrier exposes twelve response channels. Icosahedral symmetry groups them into four inequivalent bands. Complete reversible response closes their public tangent under commutators, and observer agreement requires carrier rechartings to act inside that same response. Compactness and the single fixed line then leave the Standard Model gauge Lie type. A separate matter module tests which charges coexist without anomalies; it does not turn the finite channels into observed particles.

The conditional implications share premises. Failure of the common physical tower removes the event-manifold and Einstein conclusions together. Failure of laboratory-current attachment removes the gauge and matter interpretations together. The finite normal-form, invariant-theory, and algebraic results retain their stated domains. Table 3 summarizes the principal physical interpretation claims made here.

11 Machine verification and reproducibility

Each result maps to an artifact class:

- a Lean library with no admitted propositions in the dependency closure reported here, including formal premise boundaries and countermodels;

- exact-arithmetic code receipts: integer and rational computations, interval certificates with outward rounding, and $\mathbb{F}_2$ rank computations, each emitting a canonical hashed payload;
- simulation receipt bundles pinning the applicable source or module revision, configuration, seed where stochastic, grids, tolerances, and output hashes, with deterministic reserialization distinguished from fresh source replay;
- verifiers that recompute verdicts from clause vectors and reject receipts whose stored verdict disagrees, so a caller cannot assert a truth flag directly;
- adversarial negative controls: every certificate ships with mutations that must be detected, and a control that cannot fail is treated as a defect of the certificate, not as support;
- provenance rules under which machine-readable ledgers assign every quantitative row a typed class (source result, reconstruction implication, diagnostic, registered test, or rejected candidate in the claim ledgers; tiered ancestry labels in the postdiction ledger) and reject a stronger classification unsupported by the recorded dependencies.

The repository, receipts, schemas, and rebuild instructions are public [1, 2]. The formal and executable stack checks the claims assigned to it and enforces their stated boundaries. Analytic arguments in this manuscript retain their displayed mathematical proofs, and no part of the stack establishes physical realization.

11.1 Artifact map

Table 4 identifies the principal artifacts for each result. Repository continuous integration checks the theorem-count floor, rejects admitted proofs in the public library, and rebuilds the repository-local finite certificates used here. Simulator replay is governed by the simulator receipts and reproduction commands cited below.

Table 4: Principal verification artifacts and their scope.

Result	**Principal artifacts**
Observable normalizer, stability, refinement, and complexity	`Lean/ObservableNormalForms/` and the exact proofs and reductions accompanying `extra/observable_normal_forms.tex`
Transactional diamond and consensus normal form	general local-diamond and Newman arguments proved in this manuscript; the observation-relative endpoint-uniqueness module of the Lean library; the finite transaction-model verifier with its receipt and negative controls
Event algebra, Born, Lüders, Tsirelson	`Lean/EventAlgebra/` (`Basic`, `Lueders`, `Tsirelson`, `ExpectationBound`, partition and state modules); per-module axiom audits
Central defects and edge-center entropy	overlap-cocycle and one-sided reduction proofs in this manuscript; `Lean/ObserverPatchHolography/EinsteinBranch/Entropy.lean` for scalar entropy bookkeeping after the split; `Lean/ObserverPatchHolography/CollarStates.lean` for the identity-channel model and its no-go boundary

Table 4 continued

Result	**Principal artifacts**
Port-record metric completion	`Lean/Screen/PortGramRepairBand.lean`, `Lean/Screen/PortGramRepairCovariance.lean`, and `Lean/Screen/PrimitivePortFrameQuotient.lean` for the exact repair spectrum, normalized response limit, intrinsic rank-three range, real quotient, dense integer image, and Euclidean completion; the source-pinned receipt and independent verifier under `data/repair_closure/port_gram_completion_bridge_receipt.json`
Port action, A1–A2 Lie type, and conditional matrix algebra	files under `Lean/Screen/`: `A5PortAction.lean`, `A5Commutant.lean`, `A5IncidenceResponse.lean`, `A2HolonomyBridge.lean` for the fixed-space and endogenous-transport interfaces, and `Compact12.lean` for the abstract matrix algebra; the conditional port map and A_5 covariance are checked by `code/a5_closure/port_current_inner_certificate.py`, `code/a5_closure/manifests/port_current_response_reference.json`, and `code/a5_closure/receipts/port_current_inner_reference.receipt.json`; compact-simple classification is the analytic input used in Theorem 8.2
Global form, $\mathbb{Z}_6$ kernel, and exterior selection	`Lean/Screen/Z6Exact.lean` for the abstract lattice quotient, `TraceBalancedKernel.lean` for central-parameter arithmetic, and `ExteriorSelection.lean` for the declared finite scan
Electroweak chart and strict W/Z analytic checks	`code/particles/calibration/wz_upstream_completion/outputs/certified_wz_contours.json` and `code/particles/calibration/wz_upstream_completion/outputs/certified_second_sheet_poles.json`, their schemas and fail-closed producer checks; the files certify principal-sheet zero exclusion and, for each of W and Z, one simple scalar zero with derivative and scalar-residue balls in its declared lower-half pole box on a channel-specific algebraic chart. They do not certify a physical sheet identity or independent numerical replay

Table 4 continued

Result	**Principal artifacts**
Icosahedral angular-rank theorem and primitive-port prediction	`Lean/Screen/A5PrimitivePortPrediction.lean` for equal weights, rational coefficient ratios, and refinement arithmetic, excluding the physical-sector bridge; `code/a5_fingerprint/runtime/a5_multipole_fixed_point_receipt.json`, `code/a5_fingerprint/runtime/spin_six_universality_receipt.json`, `code/a5_fingerprint/runtime/spin_six_primitive_port_prediction_receipt.json`, `code/a5_fingerprint/runtime/carrier_class_dispersion_receipt.json`, `Lean/Screen/A5CarrierClassBand.lean` for the band's rational skeleton, their mutation tests, and the custody-bound prediction register
Record-germ event-manifold gluing	analytic theorem proved in this manuscript; finite chart, transition, and signature receipts test only selected antecedents
Null tomography, small-ball arithmetic, and tensor completion	`Lean/ObserverPatchHolography/EinsteinBranch/Tensor.lean` (null directions, design/decoder, injectivity, and metric ambiguity); `Lean/docs/EINSTEIN_BRANCH_INDEX.md`; analytic determinant and norm calculations in this manuscript; explicit null-balance, Ward, Bianchi, and scale premises
Signature ladder and control	`evidence/einstein_convergence/` (four-row manifest, hashed arrays; pinned simulator revision and replay comparison procedure in its README)
Finite local source domain	staged receipts under `data/local_domain/` and the bundle verifier under `oph_fpe/local_domain/` in the simulator repository

12 Empirical meaning and falsifiability

The custody-bound register fixes target definitions, exclusion thresholds, precision floors, exposure classes, and decision rules before eligible comparison [3]. Propositions 9.4 and 9.5 are the prospective discriminators in this paper. Their coefficient rays are fixed separately, their excluded exposure classes are declared, and their dataset-specific comparisons are sealed under the conditions stated in Section 9.6. A qualifying failure rejects the tested physical propagation branch. OPH-wide scope requires a stronger source law that supplies the physical action, readout, frame transport, and exclusivity theorem for that branch.

The gravitational and gauge conclusions have separate empirical boundaries. The Einstein composition requires one physical refinement tower satisfying its event, modular, stress, entropy, continuum, vacuum, and scale premises together. The gauge and matter conclusions require laboratory-current attachment and a continuum operator limit. Failure to construct those maps leaves the corresponding finite theorems intact and removes their claimed physical application. Non-identifiability theorems determine which finite source interfaces cannot supply the missing information.

13 Discussion and comparison

Operational reconstructions derive quantum structure from information and composition principles [15, 16, 17]. OPH makes a narrower statement at the quantum stage: a completed finite record, once represented by an algebra-state pair, carries the standard event, conditioning, expectation, and correlation identities. The distinctive input is the observer-indexed normal form and its transaction contract. Unlike those reconstructions, which derive the quantum state space itself from operational axioms, OPH takes the finite algebra-state representation as an explicit input; its contribution is the completed-record surface on which the identities operate and the consensus theorem that defines completion.

Two further neighbors calibrate the record story. Quantum Darwinism locates classical objectivity in redundant environmental records accessible to many observers [31]; the completed-consensus record plays the same public role, with redundancy replaced by protected overlap agreement and an explicit repair dynamics. The cellular-automaton program reconstructs quantum mechanics from a deterministic finite substrate [32]; OPH shares the finite substrate and differs by locating the quantum identities on the completed record surface rather than in an underlying ontological basis. Wigner's-friend extensions constrain theories whose agents reason about one another's unfinished measurements [33]; the public records of this framework are post-consensus objects by construction, and the no-go premises concern pre-consensus perspectives that the completed surface does not represent.

Relational quantum mechanics treats physical facts as relational [29]; the thermal-time program reads time from modular structure [30]. OPH combines these themes by attaching public facts to protected overlap records and consuming normalized modular flow only on a typed common tower. Its local-diamond proof also has a direct distributed-systems neighbor in asynchronous agreement [26, 27].

The gravitational branch belongs to the family of thermodynamic and entanglement-based routes to geometry [40, 41, 44]. Bisognano–Wichmann modular flow and half-sided modular inclusions supply established analytic ingredients [37, 38, 39]. The contribution of Theorem 7.2 is the explicit composition contract: event, modular, stress, entropy, asymptotic, vacuum, and scale premises appear in one statement, while the finite null ambiguity identifies exactly where a metric-proportional term enters. The route from a null-projected balance and the Bianchi identity to the full equation, with Λ as an integration constant, is familiar from trace-free and unimodular formulations [42, 43]; the contribution here is the finite nine-direction design with exact determinant and decoder constants and the premise contract (G1)–(G6) stated as one implication.

The gauge section contains two reconstruction routes and one conditional global-form witness. Doplicher–Roberts/Tannaka reconstruction supplies the structural sector group [48, 49]. Independently, complete A1 response and endogenous A2 transport force the local Standard Model gauge Lie algebra. The explicit matrix map and exterior-module scan give the conditional packet group. Equality of the sector group and packet group is a separate commuting-square premise. This separation prevents the categorical reconstruction, local Lie-type theorem, and physical global group from being conflated.

For a foundations readership, the methodological claim is also concrete. Finite theorems, certified computations, reconstruction implications, and physical realization maps are different mathematical objects. Keeping those types visible makes every emergence claim auditable and gives each claim a specific failure mode. A3 also places agreement constraints before state selection. The physical adequacy of that ordering depends on the realization premises stated in Section 12.

14 Conclusion

For the specified finite observer-patch architecture, observable fibers give a canonical partial normalizer, stability moduli control approximate and refined outputs, and succinct boundary problems have explicit complexity barriers. On the transactional carrier, semantic dependency closure and revalidation yield the local diamond, so quadratic descent gives a schedule-independent public record. The completed finite algebra-state surface then carries the standard probability, conditioning, expectation, and correlation identities.

With a declared finite scheduler, the observable-normal-form interface also separates four protected first-hit failures whose cuts instantiate the behavior-cut interface exactly and transport along the stated exact finite-state morphism, as developed in the observable-normal-forms component paper [5]. These finite results supply no rates, expected hitting times, infinite-tower transport, deployed-code refinement, or physical identification.

The principal finite gauge result is the A1–A2 forcing theorem. Complete reversible response supplies a compact commutator-closed twelve-dimensional current, and endogenous overlap transport makes the proper carrier action inner on that same current. The one-dimensional fixed space excludes the centerless $\mathfrak{su}(2)^4$ alternative, leaving $\mathfrak{u}(1) \oplus \mathfrak{su}(2) \oplus \mathfrak{su}(3)$. The declared four-band map is a conditional matrix witness for this type. Trace balance and the exterior module give a conditional maximal faithful matter image with common $\mathbb{Z}_6$ kernel and an anomaly-free rank-fifteen representation; the exhaustive scan isolates its charge-conjugate pair within the stated class. The physical global quotient is not selected by this result.

The quantitative results include two prospective physical-branch predictions. The positive-chamber circulant identity forces the Koide relation exactly at tracial balance and yields a registered conditional tau test with a custody-bound kill band; the capacity identity fixes the de Sitter transfer sign; the interval-certified screen-grain root and the charged-lepton enclosures are diagnostics whose transport maps are typed as open realization maps. The source-emitted electroweak chart coordinates are not composed with the external strict one-loop fixture. The analytic receipts exclude zeros on declared principal-sheet boxes and isolate, for each of W and Z, one simple scalar zero with derivative and scalar-residue balls in its declared lower-half pole box on a channel-specific algebraic chart. They do not identify either chart with the physical resonance sheet or prove a unique continuation, full-matrix Laurent residue, or current amplitude. On the primitive twelve-port propagation branch, C_4 fixes B_0, B_6, and the complete rotated angular-rank-six template through the scale-free ratios (23). The minimal locally Lorentz-invariant Standard Model plus General Relativity predicts zero intrinsic coefficients on the stated local-vacuum baseline. The branch prediction is fixed before an eligible comparison, with its physical-sector, coherent-frame, isolation, and exclusivity premises exposed. The sixth-order isotropic term is smaller than the quartic correction by $(ak)^2/42$, so a decisive test requires sensitivity beyond a measurement of C_4 alone. The source-seam branch fixes the distinct edge-orbit ratios in (26), with a negative rank-six coefficient and a source-derived internal translation action. Its promotion requires the physical position, field, clock, scale, frame, and readout maps. Theorem 9.6 binds the two frozen rays and every member of its declared positive-weight scalar cosine class into one surface. That class shares $C_4 < 0$ with the isotropic floor $B_0/C_4^2 \geq 10/21$, saturated exactly by single-radius carriers, and confines the rank-six-to-isotropic ratio to the exact band $[-16/135, 16/75]$. A resolved dispersion measurement outside that surface excludes the declared class. The verdict is broader than a single-branch exclusion. No comparison-ready dataset is asserted.

The geometry and dynamics results have a different status. A Lorentz-group action and its hyperbolic homogeneous space follow from the oriented spherical support; their physical interpretation requires event, clock, and frame maps. A Lorentzian event manifold follows only under the displayed event premises. The Einstein field equation follows only from one common tower satisfying the modular, stress, entropy, asymptotic, vacuum, and scale premises. The physical tower, continuum limit, laboratory-current attachment, quantum-field realization, and the listed interpretation maps are the open realization maps of the program. The paper therefore offers a finite reconstruction framework with exact results and explicit physical hypotheses; it does not claim a completed derivation of all physics, particle masses, a cosmology, or a proof that the simulator describes our universe. Its closure hypothesis, three axioms, named dynamics, finite consequences, and physical realization maps are distinct parts of one logical construction.

The ambition of the program is stated here at full strength, in its proper place after the results and their boundaries. The goal is a complete description of physical reality derived from consistency requirements alone: every dimensionless constant, every structural property, and every dynamical law recovered as the content of one self-consistent timeless structure, with no free parameter and no external input. On the closure hypothesis of Section 1.3, that goal addresses two classical questions at once. The answer offered to why anything exists is that a timeless self-referential structure requires no external cause: existence coincides with self-consistency, and observers arise downstream inside the structure as the systems that recover and construct its operative simulator specification in their subjective time. The axioms state one finite way for that reconstruction-and-construction loop to close consistently. They require no a priori derivation from an earlier layer of reality. The answer offered to why exactly this world is that the consistency requirements are far more selective than they first appear: the results assembled here force the local Standard Model gauge type from twelve ports and two axioms, fix the unique rank-six invariant line of the carrier, give a rigid fingerprint on the named vertex-orbit branch, fix the Koide relation exactly at the certified balance, and pose exact closure equations for the fine-structure coupling and the cosmological capacity whose displayed candidates stand within 2.5×10^{-6} and one percent of the measured coordinates as typed diagnostics.

The distance between that ambition and the present results is recorded in this paper's own typing rather than left to interpretation. The finite theorems are proved; the quantitative closures are typed as diagnostics or registered tests; the physical realization maps are named, with no construction asserted; and the frozen branch predictions carry separate scale-free coefficient relations and registered kill rules. The program therefore submits its ambition in the only scientifically admissible form: a sequence of exact claims each of which can fail, a registered fingerprint that experiments can match or refute, and a boundary ledger stating exactly what is unproved. Whether the full description exists is undecided; the contribution of this paper is to make the question precise, its completed stages machine-audited, and its registered surfaces falsifiable.

Reproducibility statement

All theorems, certificates, simulation configurations, receipts, and the formal library are public in the project repositories [1, 2]. The paper release manifest records the release identifier (r2020) together with PDF hashes and sizes. Computational receipt bundles pin the applicable source or module revision, configuration, seed where stochastic, and output hashes. Clean-checkout rebuild instructions and negative controls accompany the artifacts. The claim ledger and prediction register are machine readable [3].

The Lean source tree, pinned toolchain, exact verifiers, manifests, receipts, negative controls, tests, finite signature ladder, and local-domain evidence are identified in Table 4. The complete simulation source is maintained in the public simulator repository at the evidence-producing revisions [2].

This manuscript synthesizes and strengthens results whose longer proofs also appear in public component preprints on observable normal forms, finite event algebras, consensus, Einstein reconstruction, and gauge structure [5, 6, 7, 10, 11]. The component relationship is disclosed here so that overlap is visible.

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Author contributions. Bernhard Mueller developed the OPH concept and mathematical synthesis, wrote the main manuscript, and coordinated the formal and computational evidence. Alexander Osika contributed the physical-realization and hardware boundary, prototype framing, and manuscript review. Mario Poneder contributed the finite port-current construction and verifier, gauge and matter certificates, and proof review of the edge-entropy and Einstein-normalization branches. Kai Xue contributed consistency checks, simulator review, visualization review, and prototype-engineering review. Ben Cassie contributed implementation-claim and physical-evidence review. Peter Nguyen contributed proof auditing, branch-boundary analysis, compact-gauge and global-quotient analysis, and manuscript consistency review. Jinwook Kim contributed the Lean formalization of protected-behavior obstruction profiles and the conditional mechanism-variant example in the observable normal forms library, their integration into the paper surfaces, and the registered claim gates and reproducible-build reliability work. David Matscheko contributed proof review of consensus repair, observable normal forms, modular and Einstein algebra, edge entropy, hypercharge, and the $\mathbb{Z}_6$ quotient. Jonathan Hill contributed Lean formalization and proof auditing of observable normal forms, refinement and repair results, complexity classifications, and artifact coverage. William T. Glynn contributed simulator development, computational implementation, reproducible-build infrastructure, and release-validation work in the main OPH research repository. Maarten Antoine Visser contributed consensus-protocol review and physics feedback on the foundations and emergent-spacetime framing. Kale Arnav Anirudha contributed consensus-paper material, asynchronous-agreement analysis, repair-map definitions, and formal review of the protocol surface. All authors reviewed and approved the manuscript. All authors agree to be accountable for all aspects of the work.
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Data availability. The finite witnesses, simulation configurations, receipts, claim ledgers, and release manifests supporting this study are available in the public OPH and simulation repositories [1, 2]. Individual receipts and their bundle documentation provide the source and output provenance for computational claims. The artifact map identifies the finite signature-ladder and local-domain evidence cited in the manuscript.
Code availability. The Lean sources, exact-arithmetic verifiers, finite scans, simulation code, tests, and rebuild instructions are available in the same public repositories [1, 2]. The computational claims in this manuscript identify their principal files in Section 11.
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